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SUPEREMBEDDING APPROACH and S-DUALITY.
A unified description of superstring and super-D1-brane.

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Abstract

It is proved that a basic superembedding equation for the 2-dimensional worldsheet superspace $\Sigma^{(2|8+8)}$ embedded into D=10, type IIB superspace $\underline{\mathcal{M}}^{(10|16+16)}$ provides a universal, S-duality invariant description of a fundamental superstring and super-D1-brane. We work out generalized action principle, obtain superfield equations of motion for both these objects and find how the S-duality transformations relate the superfield equations of superstring and super-D1-brane.

The superembedding of 6-dimensional worldsheet superspace $\Sigma^{(6|16)}$ into the D=10, type IIB superspace $\underline{\mathcal{M}}^{(10|16+16)}$ will probably provide a similar universal description for the set of type IIB super–NS5–brane, super-D5-brane and a Kaluza-Klein monopole (super-KK5-brane).

1 Introduction

It is believed that type IIB superstring theory has $SL(2,Z)$ S-duality symmetry [1, 2]. $SL(2,R)$ symmetry had been found in the low energy limit provided by type IIB $D=10$ supergravity theory [3]. When nonperturbative BPS states including super-Dp-branes with $p=1,3,5,7,9$ are considered, the Dirac charge quantization condition for the brane charges shall be taken into account. It reduces $SL(2,R)$ down to $SL(2,Z)$ which, thus, appears as a quantum S-duality group.

On the classical level, however, one can consider the continuous group $SL(2,R)$. When superspace is flat this symmetry reduces down to $SO(2)$ acting on Grassmannian coordinates of the flat superspace.

Among the BPS states are ones which are invariant under S-duality. A well known example is super–D3–brane [4, 5]. The analysis of the supergravity solution [6] and of the action in the second order approximation [7] indicate that Kaluza–Klein monopole 5-brane (super–KK5–brane) should be invariant as well. In distinction, the fundamental string and super-D1-brane as well as the type IIB NS5-brane and super-D5-brane are not invariant under $SL(2,Z)$ group. The reason is that fundamental string is coupled minimally to the NS–NS (Neveu-Schwarz–Neveu-Schwarz) 2-form gauge field B_2 of type IIB supergravity, while super-D1-brane is coupled to the RR (Ramond-Ramond) gauge field C_2. The type IIB NS5 superbrane and super-D5-brane carry unit magnetic charges with respect to B_2 and C_2 fields. The classical S-duality symmetry $SL(2,R)$ mixes B_2 and C_2 fields as well as RR and NS-NS charges.

The minimal irreducible multiplets of BPS state under the S-duality group $SL(2,Z)$ are provided by families of so–called (p,q) strings ($(1,0)$ corresponds to the fundamental superstring and $(0,1)$ to the super-D1-brane) [8, 9] and (p,q) 5-branes [10]. An intensive search for an S-duality invariant universal description of $SL(2,Z)$ multiplets of BPS states can be witnessed today [11, 12, 13, 14].

The Hamiltonian analysis and some related studies [5, 11] indicate that the (p,q) string can be associated with D1-brane action considered on the surface of Born–Infeld equation. Thus, a unified description of (p,q)–string is basically a *universal description of fundamental superstring (super–NS1–brane) and Dirichlet superstring (super–D1–brane).*

The main message of this article is that such a universal, S-duality invariant description of the sets of superbranes is provided by the superembedding approach [15, 16, 17, 19, 20, 21, 22], [23, 24, 25, 26, 27, 28]. In this framework a 10-dimensional type IIB super-(D)p-brane is described by a *worldsheet superspace* $\Sigma^{(p+1|16)}$ embedded into the target type IIB superspace $\underline{\mathcal{M}}^{(10|32)}$. The number of fermionic 'directions' of the worldsheet superspace ($=16$) is twice less than the one of the target superspace ($=32$). And each of the fermionic directions is in one–to–one correspondence [15] with a parameter of κ-symmetry [29] inherent to super-p-brane actions [1] or, equivalently, with a parameter of supersymmetry preserved by the BPS–state corresponding to the super-(D)p-brane.

The embedding is specified by the so–called geometrodynamical constraint or *basic superembedding equation*. Its form is universal: the superembedding condition implies that a pull–back of a bosonic supervielbein form of the target superspace on the worldvol-

[1]Note that there exists a possibility to introduce a worldsheet superspace with less number n of the fermionic dimensions $\Sigma^{(p+1|n)}$, $n\leq 8$. Then the remaining $(16-n)$ κ-symmetries become nonmanifest gauge symmetries of a superfield action, which can be built in this case [17, 18, 30, 28].

ume superspace has vanishing projections on the fermionic worldvolume directions (see [15, 16, 19, 23, 25, 26, 28] and below). For branes in $D=10$ type II superspaces the superembedding equation contains dynamical equations of motion among its consequences [21, 23, 24, 25]. Then the universal description of the superstring and super-D1-brane as well as of the set of super-D5-brane, type IIB super–NS5–brane and super-KK5-brane should occur if i) the natural worldvolume superspaces of these objects are of the same type: i.e. have the same number of bosonic and fermionic directions and same worldvolume 'quantum numbers' of the fermionic coordinates, ii) the equations of motion for all objects can be encoded in the same superembedding condition. The former item can be justified by studying the structure of the κ–symmetry of the superbranes.

An investigation of the universal description of different branes in the frame of superembedding approach will hopefully provide us with new insights in the structure of String/M–theory. In this paper we begin the above program by studying the universal, S-duality invariant description of superstring and super-D1-brane.

1.1 Basic notations and short summary

The basic equation of superembedding approach for the type IIB superstring specifies the embedding

$$Z^{\underline{M}} = \hat{Z}^{\underline{M}}(\zeta): \qquad X^{\underline{m}} = \hat{X}^{\underline{m}}(\xi,\eta), \quad \Theta^{\underline{\mu}1} = \hat{\Theta}^{\underline{\mu}1}(\xi,\eta), \quad \Theta^{\underline{\mu}2} = \hat{\Theta}^{\underline{\mu}2}(\xi,\eta) \tag{1.1}$$

of 2-dimensional worldsheet superspace

$$\Sigma^{(2|8+8)}: \ \{(\zeta^M)\} = \{(\xi^m, \eta^{+q}, \eta^{-\dot{q}})\}, \quad m=0,1, \quad q=1,\ldots,8, \ \ \dot{q}=1,\ldots,8 \tag{1.2}$$

into the 10-dimensional type IIB superspace

$$\underline{\mathcal{M}}^{(10|16+16)} = \{(X^{\underline{m}}, \Theta^{\underline{\mu}1}, \Theta^{\underline{\mu}2})\}, \qquad \underline{m}=0,\ldots,9, \quad \underline{\mu}=1,\ldots,16. \tag{1.3}$$

It can be written in the form (see [15, 19, 20, 22, 23, 28] and refs. in [28])

$$\hat{E}^{\underline{a}}_{+q}\left(\hat{Z}(\zeta)\right) = 0, \qquad \hat{E}^{\underline{a}}_{-\dot{q}}\left(\hat{Z}(\zeta)\right) = 0. \tag{1.4}$$

Here $E^{\underline{a}} \equiv dZ^{\underline{M}} E_{\underline{M}}^{\ \underline{a}}(Z)$ is the bosonic supervielbein form of the type IIB superspace and

$$\hat{E}^{\underline{a}} \equiv dZ^{\underline{M}}(\zeta)\hat{E}_{\underline{M}}^{\ \underline{a}}\left(\hat{Z}(\zeta)\right) = e^{++}\hat{E}_{++}^{\ \underline{a}} + e^{--}\hat{E}_{--}^{\ \underline{a}} + e^{+q}\hat{E}_{+q}^{\ \underline{a}} + e^{-\dot{q}}\hat{E}_{-\dot{q}}^{\ \underline{a}} \tag{1.5}$$

is its pull-back on the worldsheet superspace $\Sigma^{(2|8+8)}$. In Eq. (1.5)

$$e^A \equiv d\zeta^M e_M^{\ A}(\zeta) = \left(e^{++}, e^{--}; e^{+q}, e^{-\dot{q}}\right) \tag{1.6}$$

is an intrinsic supervielbein of the worldsheet superspace. It can be either subject to the supergravity constraints, or induced by the embedding [23, 24].

When the superembedding equations (1.4) are taken into account, the general decomposition of the pull-back of the bosonic vielbein form (1.5) reduces to

$$\hat{E}^{\underline{a}} \equiv dZ^{\underline{M}}(\zeta)\hat{E}_{\underline{M}}^{\ \underline{a}}\left(\hat{Z}^{\underline{M}}(\zeta)\right) = e^{++}\hat{E}_{++}^{\ \underline{a}} + e^{--}\hat{E}_{--}^{\ \underline{a}}. \tag{1.7}$$

The superembedding equation for type IIB superstring puts the theory on the mass shell as it does for all the objects with more then 16 target space supersymmetries, such as D=11 supermembrane [23] (super-M2-brane), super-M5-brane [25] and type IIA superstring [21, 23]. However, as it was found in [23], the case of type IIB has a peculiarity. Namely, the equations of motion which follow from (1.7) contains a constant parameter a (see also [21] for $D = 3$). E.g., for the case of *flat target type IIB superspace*

$$E^{\underline{a}} = \Pi^{\underline{m}} \delta_{\underline{m}}^{\underline{a}}, \qquad \Pi^{\underline{m}} = dX^{\underline{m}} - id\Theta^{1\underline{\mu}} \sigma^{\underline{m}}_{\underline{\mu\nu}} \Theta^{1\underline{\nu}} - id\Theta^{2\underline{\mu}} \sigma^{\underline{m}}_{\underline{\mu\nu}} \Theta^{2\underline{\nu}}, \tag{1.8}$$

$$E^{\underline{\alpha}1} = d\Theta^{1\underline{\mu}} \delta_{\underline{\mu}}^{\underline{\alpha}}, \qquad E^{\underline{\alpha}2} = d\Theta^{2\underline{\mu}} \delta_{\underline{\mu}}^{\underline{\alpha}} \tag{1.9}$$

the fermionic equations which follow from (1.7) can be written in the form

$$\left(\nabla_{--}\hat{\Theta}^{\underline{\mu}1} + a\nabla_{--}\hat{\Theta}^{\underline{\mu}2}\right) \sigma_{\underline{a\mu\nu}} \hat{E}^{\underline{a}}_{++} = 0, \qquad \left(\nabla_{++}\hat{\Theta}^{\underline{\mu}2} - a\nabla_{++}\hat{\Theta}^{\underline{\mu}1}\right) \sigma_{\underline{a\mu\nu}} \hat{E}^{\underline{a}}_{--} = 0, \tag{1.10}$$

while the fermionic equations for type IIB superstring are

$$\nabla_{--}\hat{\Theta}^{\underline{\mu}1} \sigma_{\underline{a\mu\nu}} \hat{E}^{\underline{a}}_{++} = 0, \qquad \nabla_{++}\hat{\Theta}^{\underline{\mu}2} \sigma_{\underline{a\mu\nu}} \hat{E}^{\underline{a}}_{--} = 0 \tag{1.11}$$

Eq. (1.11) coincides with (1.10) only when the constant a vanishes.

On the other hand, if one uses an evident scale invariance of Eqs. (1.10) and multiplies them by $\frac{1}{\sqrt{1+a^2}}$, then he arrives at the equations related to Eqs. (1.11) by the $SO(2)$ transformation of the Grassmann coordinates of the flat type IIB superspace [23]

$$\begin{pmatrix} \hat{\Theta}^{\underline{\mu}1\prime} \\ \hat{\Theta}^{\underline{\mu}2\prime} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{1+a^2}} & \frac{a}{\sqrt{1+a^2}} \\ -\frac{a}{\sqrt{1+a^2}} & \frac{1}{\sqrt{1+a^2}} \end{pmatrix} \begin{pmatrix} \hat{\Theta}^{\underline{\mu}1} \\ \hat{\Theta}^{\underline{\mu}2} \end{pmatrix} \equiv \begin{pmatrix} Cos\ \alpha & Sin\ \alpha \\ -Sin\ \alpha & Cos\ \alpha \end{pmatrix} \begin{pmatrix} \hat{\Theta}^{\underline{\mu}1} \\ \hat{\Theta}^{\underline{\mu}2} \end{pmatrix} \tag{1.12}$$

The $SO(2)$ transformations (1.12) are not a symmetry of the action of type IIB superstring. They are the *flat superspace image of the S-duality group* $SL(2, R)$. (We will make it apparent in Section 6). Nevertheless, $SO(2)$ *is* an evident symmetry of the superembedding equation (1.4), (1.7).

Thus the solutions of the superembedding equation (1.10) with $a \neq 0$, which are characterized by a superfield generalization of Eqs. (1.11) (see below), shall describe extended objects which are related to fundamental superstring by S-duality. As we will demonstrate in this paper, Eqs. (1.10) with $a \neq 0$ are just the fermionic equations of motion for a super-D1-brane. The numerical parameter a is expressed through the on-shell value of the gauge field strength of the super-D1-brane $dA - B_2 = 1/2e^{++} \wedge e^{--} F^{(0)}$

$$a = \pm\sqrt{\frac{1 + F^{(0)}}{1 - F^{(0)}}}, \qquad dF^{(0)} = 0. \tag{1.13}$$

The 'scale' $\sqrt{1 + a^2}$ of the parameter a is unessential when the equations are considered, while the $SO(2)$ parameter (2α) is essentially in one–to–one correspondence with the on–shell value of the field strength

$$Cos\ 2\alpha = -F^{(0)}, \qquad Cos\ \alpha \equiv \frac{a}{\sqrt{1 + a^2}} = \pm\sqrt{\frac{1 + F^{(0)}}{2}}. \tag{1.14}$$

Sections 2–5 of the present paper are devoted to the proof of the above result. We motivate that the worldvolume superspace of a super-D1-brane is of the same type as the one of the fundamental superstring (1.2) and that the superembedding condition is the same as well (1.4). We obtain the explicit relation (1.13) between the parameter a and on-shell value of the generalized field strength of the worldvolume gauge field and justify the statement that the value of the field strength, the superembedding equation and the fermionic superfield equations specify the description of the super-D1-brane completely.

To this end we use the generalized action principle [24, 26, 32] for superstring and super-D1-brane. It is a brane counterpart of the group manifold action for supergravity [33]. The advantage of the generalized action [24] is that it produces the superembedding equations, (a superfield generalization of the) proper equations of motion and the worldvolume supergravity constraints in a universal manner. Moreover, it possesses a generalized κ–symmetry [29] and, hence, provides a bridge between the standard (Green-Schwarz—Dirac-Born-Infeld) formulation and the superembedding approach [23, 25, 28]. The κ–symmetry appears in the irreducible form and, thus, can be used to determine the properties of the worldsheet superspace necessary for the superembedding description of the superbranes.

1.2 Contents of the paper

For simplicity, in Sections 2,3,4,5 we restrict ourself by the case of flat target superspace, where the classical S-duality group $SL(2,R)$ reduces to $SO(2)$. Section 2 is devoted to the general solution of the superembedding equation. Lorentz harmonics (moving frame variables) are introduced here. In Section 3 and 4 we elaborate generalized action principle and obtain superfield equations of motion for superstring and super–D1–brane. The universal description of the superstring and super–D1–brane in the frame of superembedding approach is considered in Section 5. The results for general supergravity background are presented in Section 6. It is demonstrated that the $SO(2)$ rotations which, together with super–Weyl transformations, relate super–D1–brane and superstring, are indeed the compensated rotations which appear when the classical S–duality group $SL(2,R)$ acts on the unimodular matrix constructed from the axion and dilaton superfields. We also use the super–Weyl transformations to derive the expression for (p,q)–string tension [8]. Some useful formulae are collected in Appendices.

2 General solution of superembedding equation. The role of Lorentz harmonics.

2.1 Lorentz harmonics and superembedding equation

To obtain in a regular manner all the consequences of the superembedding equation (1.4) it is convenient to use its equivalent form [23] (see Appendix A)

$$\hat{E}^I \equiv \hat{E}^{\underline{b}} U_{\underline{b}}{}^I = 0, \qquad I = 1, \dots 8, \tag{2.1}$$

where $U_{\underline{a}}{}^{I}$ are 8 orthogonal and normalized 10-vector superfields

$$U^{\underline{a}I}U^{J}_{\underline{a}} = -\delta^{IJ}. \tag{2.2}$$

Their set can be completed up to a basis of 10-dimensional (tangent) space, or, equivalently, to the Lorentz group valued matrix (*moving frame matrix*)

$$U^{(\underline{b})}_{\underline{a}} = \left(U^{0}_{\underline{a}}, U_{\underline{a}}{}^{J}, U^{9}_{\underline{a}}\right) \in SO(1,9) \quad \Leftrightarrow \quad U^{(\underline{b})}_{\underline{a}} U^{\underline{a}(\underline{c})} = \eta^{(\underline{b})(\underline{c})} \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\delta^{IJ} & 0 \\ 0 & 0 & -1 \end{pmatrix} \tag{2.3}$$

by adding 2 orthogonal and normalized vectors $U^{0}_{\underline{a}}$, $U^{9}_{\underline{a}}$

$$U^{\underline{a}0}U^{0}_{\underline{a}} = 1, \quad U^{\underline{a}0}U^{9}_{\underline{a}} = 0, \quad U^{\underline{a}9}U^{9}_{\underline{a}} = -1. \qquad U^{\underline{a}0}U^{I}_{\underline{a}} = 0 = U^{\underline{a}9}U^{I}_{\underline{a}}. \tag{2.4}$$

It is convenient to replace the vectors $U^{0}_{\underline{a}}$, $U^{9}_{\underline{a}}$ by their light–like combinations

$$U^{++}_{\underline{a}} = U^{0}_{\underline{a}} + U^{9}_{\underline{a}}, \qquad U^{--}_{\underline{a}} = U^{0}_{\underline{a}} - U^{9}_{\underline{a}} \tag{2.5}$$

$$U^{++}_{\underline{a}}U^{++\underline{a}} = 0, \qquad U^{--}_{\underline{a}}U^{--\underline{a}} = 0, \qquad U^{++}_{\underline{a}}U^{--\underline{a}} = 2.$$

and to define the moving frame matrix (2.3) by

$$U^{(\underline{b})}_{\underline{a}} = \left(U^{++}_{\underline{a}}, U^{--}_{\underline{a}}, U_{\underline{a}}{}^{J}\right) \in SO(1,9) \quad \Leftrightarrow \quad U^{(\underline{b})}_{\underline{a}} U^{\underline{a}(\underline{c})} = \eta^{(\underline{b})(\underline{c})} \equiv \begin{pmatrix} 0 & 2 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & -\delta^{IJ} \end{pmatrix} \tag{2.6}$$

Its elements can be recognized as *Lorentz harmonics*, introduced by Sokatchev [34].

The Lorentz rotation by matrix (2.6) provides us with the bosonic supervielbein forms

$$\hat{E}^{(\underline{a}')} \equiv \hat{E}^{\underline{b}}U_{\underline{b}}{}^{(\underline{a}')} = \left(\hat{E}^{++}, \hat{E}^{--}, \hat{E}^{I}\right), \qquad \hat{E}^{\pm\pm} = \hat{E}^{\underline{a}}U^{\pm\pm}_{\underline{b}}, \quad \hat{E}^{I} = \hat{E}^{\underline{a}}U^{I}_{\underline{b}} \tag{2.7}$$

adapted to the superembedding in the sense of Eq. (2.1). To adapt the complete supervielbein (more precisely, the pull-back of the supervielbein) to the superembedding, we need in the double covering of the Lorentz group valued matrix U (2.6). It is given by 16×16 matrix

$$V^{(\underline{\alpha}')}_{\underline{\alpha}} = \left(V_{\underline{\alpha}q}{}^{+}, V_{\underline{\alpha}\dot{q}}{}^{-}\right)^{T} \quad \in \quad Spin(1,9) \tag{2.8}$$

whose rectangular 16×8 blocks $V^{+q}_{\underline{\alpha}}, V^{-\dot{q}}_{\underline{\alpha}}$ ($q = 1, \ldots 8$, $\dot{q} = 1, \ldots 8$) are called *spinor Lorentz harmonics* [35, 36, 37, 23, 24] or spinor moving frame variables [37]. The claimed statement that (2.8) provides a double covering of the Lorentz rotation described by (2.6) can be expressed by the conditions of γ-matrix preservation [2]

$$U^{(\underline{a})}_{\underline{b}}\sigma^{\underline{b}}_{\underline{\alpha}\underline{\beta}} = V_{\underline{\alpha}}{}^{(\underline{\gamma})}\sigma^{(\underline{a})}_{(\underline{\gamma})(\underline{\delta})}V^{(\underline{\delta})}_{\underline{\beta}}, \tag{2.9}$$

With an appropriate $SO(1,1)\otimes SO(8)$ invariant representation for $D=10$ σ-matrices, the constraints (2.9) can be split into the set of the following covariant relations

$$U^{++}_{\underline{a}}\sigma^{\underline{a}}_{\underline{\alpha}\underline{\beta}} = 2V_{\underline{\alpha}q}{}^{+}V_{\underline{\beta}q}{}^{+}, \qquad U^{++}_{\underline{a}}\tilde{\sigma}^{\underline{a}\underline{\alpha}\underline{\beta}} = 2V^{+\underline{\alpha}}_{\dot{q}}V^{+\underline{\beta}}_{\dot{q}}, \tag{2.10}$$

[2]The constraints (2.9) are certainly reducible. However, they are most convenient for calculations. The irreducible form of the constraints for spinor harmonics can be found in [36, 37].

$$U^{--}_{\underline{a}}\sigma^{\underline{a}}_{\underline{\alpha\beta}} = 2V^{\ -}_{\underline{\alpha}\dot{q}}V^{\ -}_{\underline{\beta}\dot{q}}, \qquad U^{--}_{\underline{a}}\tilde{\sigma}^{\underline{a\alpha\beta}} = 2V^{-\underline{\alpha}}_{q}V^{-\underline{\beta}}_{q}, \tag{2.11}$$

$$U^{I}_{\underline{a}}\sigma^{\underline{a}}_{\underline{\alpha\beta}} = 2V^{\ +}_{(\underline{\alpha}q}\gamma^{I}_{q\dot{q}}V^{\ -}_{\underline{\beta})\dot{q}}, \qquad U^{I}_{\underline{a}}\tilde{\sigma}^{\underline{a\alpha\beta}} = -2V^{-(\underline{\alpha}}_{q}\gamma^{I}_{q\dot{q}}V^{+\underline{\beta})}_{\dot{q}}, \tag{2.12}$$

which imply, in particular, that the spinor harmonics $V^{\ +q}_{\underline{\alpha}}, V^{\ -\dot{q}}_{\underline{\alpha}}$ can be treated as square roots from the light–like vectors $U^{++}_{\underline{a}}, U^{--}_{\underline{a}}$.

The second equalities in Eqs. (2.10), (2.11), (2.12) involve the inverse Lorentz harmonics [37]

$$V^{-\underline{\alpha}}_{p}V^{\ +}_{\underline{\alpha}q} = \delta_{pq}, \qquad V^{-\underline{\alpha}}_{p}V^{\ -}_{\underline{\alpha}\dot{q}} = 0, \tag{2.13}$$
$$V^{+\underline{\alpha}}_{\dot{p}}V^{\ +}_{\underline{\alpha}q} = 0, \qquad V^{+\underline{\alpha}}_{\dot{p}}V^{\ -}_{\underline{\alpha}\dot{q}} = \delta_{\dot{p}\dot{q}}.$$

The supervielbein adapted for the superembedding in the sense of Eq. (2.1) is

$$(\hat{E}^{(\underline{a}^{\prime})};\ \ \hat{E}^{(\underline{\alpha}^{\prime})1},\ \ \hat{E}^{(\underline{\alpha}^{\prime})2}) = \left(\hat{E}^{++},\hat{E}^{--},\hat{E}^{I};\ \hat{E}^{+q1},\hat{E}^{-\dot{q}1},\ \hat{E}^{+q2},\hat{E}^{-\dot{q}2}\right), \tag{2.14}$$

where

$$\hat{E}^{+q1,2} = \hat{E}^{\underline{\alpha}1,2}V^{\ +}_{\underline{\alpha}q}, \qquad \hat{E}^{-\dot{q}1,2} = \hat{E}^{\underline{\alpha}1,2}V^{\ -}_{\underline{\alpha}\dot{q}}. \tag{2.15}$$

For the case of flat type IIB superspace Eqs. (2.15) become

$$\hat{E}^{+q1,2} = d\hat{\Theta}^{\underline{\mu}1,2}V^{\ +}_{\underline{\mu}q}, \qquad \hat{E}^{-\dot{q}1,2} = d\hat{\Theta}^{\underline{\mu}1,2}V^{\ -}_{\underline{\mu}\dot{q}}. \tag{2.16}$$

The geometry of the worldsheet superspace can be specified by embedding in the following sense [24]. One can choose the bosonic components of the worldsheet supervielbein (1.6) to be equal to the $E^{\pm\pm}$ components of the adapted supervielbein (2.14) (see Appendix A)

$$\hat{E}^{++} \equiv \hat{E}^{\underline{b}}U^{++}_{\underline{b}} = e^{++}, \qquad \hat{E}^{--} \equiv \hat{E}^{\underline{b}}U^{--}_{\underline{b}} = e^{--}. \tag{2.17}$$

The fermionic worldsheet supervielbein forms can be identified with 16 linear combinations of the pull-backs of target superspace fermionic forms (2.15). We make the choice

$$e^{+q} = E^{+q1} = \hat{E}^{\underline{\alpha}1}V^{\ +q}_{\underline{\alpha}}, \qquad e^{-\dot{q}} = \hat{E}^{-\dot{q}2} = \hat{E}^{\underline{\alpha}2}V^{\ -\dot{q}}_{\underline{\alpha}}, \tag{2.18}$$

which will be motivated below by a study of the irreducible κ–symmetry. In the case of *flat type IIB superspace* the worldsheet spin connections and $SO(8)$ gauge field can be identified with Cartan forms [37, 23, 24]

$$\omega \equiv \frac{1}{2}U^{--}_{\underline{a}}dU^{++\underline{a}}, \qquad A^{IJ} \equiv U^{I}_{\underline{a}}dU^{J\underline{a}} \tag{2.19}$$

while the remaining Cartan forms

$$f^{\pm\pm I} \equiv U^{\pm\pm}_{\underline{a}}dU^{I\underline{a}} \tag{2.20}$$

are covariant with respect to local $SO(1,1)\times SO(8)$ transformations [3]. This $SO(1,1)\times SO(8)$ is an evident symmetry of the relations (2.1), (2.17), (2.18) which acts on the

[3]In general supergravity background one can define the worldsheet connections and covariant forms by adding to the expressions (2.19), (2.20) the supergravity spin connection $w_{\underline{a}}^{\ \underline{b}}$ contracted with harmonic vectors

$$\tilde{f}^{\pm\pm I} \equiv f^{\pm\pm I}+(UwU)^{\pm\pm I} \equiv U^{\pm\pm}_{\underline{a}}(dU^{I\underline{a}}+w_{\underline{a}}^{\ \underline{b}}U^{I}_{\underline{b}}), \qquad \tilde{\omega} \equiv \omega+\frac{1}{2}(UwU)^{--|++}, \qquad \tilde{A}^{IJ} = A^{IJ}+(UwU)^{IJ}$$

Actually, this is a prescription for the construction of the 'gauge fields of nonlinear realization' [38].

bosonic worldsheet supervielbein and vector harmonics as follows

$$e^{\pm\pm\prime} = e^{\pm\pm} exp(\pm 2\beta(\zeta)), \quad U_{\underline{a}}^{\pm\pm\prime} = U_{\underline{a}}^{\pm\pm} exp(2\beta(\zeta)), \tag{2.21}$$

$$U_{\underline{a}}^{I} \to U_{\underline{a}}^{I\prime} = U_{\underline{a}}^{J} \mathcal{O}^{JI}, \qquad \mathcal{O}^{JK}\mathcal{O}^{IK} = \delta^{IJ}.$$

Such symmetry makes possible to consider the vectors $U_{\underline{a}}^{\pm\pm}, U_{\underline{a}}^{I}$ (constrained by (2.6)) as homogeneous coordinates of the coset

$$\frac{SO(1,9)}{SO(1,1)\otimes SO(8)} = \left\{ (U_{\underline{a}}^{\pm\pm}, U_{\underline{a}}^{I}) \right\} \tag{2.22}$$

and to identify them with Lorentz harmonic variables [34].

The derivatives of the harmonic variables, which do not break the constraints (2.6) are expressed through the Cartan forms

$$\mathcal{D}U_{\underline{a}}^{++} \equiv dU_{\underline{a}}^{++} - U_{\underline{a}}^{++}\omega = U_{\underline{a}}^{I} f^{++I}, \qquad \mathcal{D}U_{\underline{a}}^{--} \equiv dU_{\underline{a}}^{--} + U_{\underline{a}}^{--}\omega = U_{\underline{a}}^{I} f^{--I}, \tag{2.23}$$

$$\mathcal{D}U_{\underline{a}}^{I} \equiv dU_{\underline{a}}^{I} + U_{\underline{a}}^{J} A^{JI} = \frac{1}{2} U_{\underline{a}}^{--} f^{++I} + \frac{1}{2} U_{\underline{a}}^{++} f^{--I}. \tag{2.24}$$

2.2 General solution of superembedding equation in flat type IIB superspace.

To investigate the consequences of the superembedding equation (2.1) one can study its integrability conditions [23]

$$\hat{E}^{I} = 0 \qquad \Rightarrow \qquad d\hat{E}^{I} \equiv d\hat{E}^{\underline{a}} U_{\underline{a}}^{I}(\zeta) + \hat{E}^{\underline{a}} \wedge dU_{\underline{a}}^{I}(\zeta) = 0. \tag{2.25}$$

The integrability conditions for 'conventional constraints' (2.17), (2.18) should determine the worldsheet torsion forms.

In the flat superspace Eq. (2.25) acquires the form

$$\mathcal{D}\hat{E}^{I} \equiv d\hat{E}^{I} + \hat{E}^{J} \wedge A^{IJ} = T^{\underline{a}} U_{\underline{a}}{}^{I} + \frac{1}{2}\hat{E}^{++} \wedge f^{--I} + \frac{1}{2}\hat{E}^{--} \wedge f^{++I} = \tag{2.26}$$

$$= -i\left(e^{+q} \wedge \hat{E}^{-\dot{q}1} - ie^{-\dot{q}} \wedge \hat{E}^{+q2}\right)\gamma^{I}_{q\dot{q}} + \frac{1}{2}\hat{E}^{++} \wedge f^{--I} + \frac{1}{2}\hat{E}^{--} \wedge f^{++I} = 0.$$

Note that the same equation occurs in curved type IIB superspace with the standard choice of torsion constraints

$$T^{\underline{a}} \equiv \mathcal{D}E^{\underline{a}} \equiv dE^{\underline{a}} - E^{\underline{b}} \wedge w_{\underline{b}}{}^{\underline{a}} = -i(E^{\underline{\alpha}1} \wedge E^{\underline{\beta}1} + E^{\underline{\alpha}2} \wedge E^{\underline{\beta}2})\sigma^{\underline{a}}_{\underline{\alpha}\underline{\beta}},$$

but with covariantized Cartan forms defined in Appendix A ($f^{\pm\pm I} \to f^{\pm\pm I} + U^{\pm\pm}wU^{I}$, see also footnote 3).

Substituting the most general expression for the pull-backs of the fermionic supervielbein forms $\hat{E}^{-\dot{q}1}$ and $\hat{E}^{+q2}$

$$\hat{E}^{-\dot{q}1} = e^{+p}\chi^{--}_{p\dot{q}} + e^{-\dot{p}}h_{\dot{p}}^{-\dot{q}} + e^{\pm\pm}\Psi^{-}_{\pm\pm\dot{q}}, \qquad \hat{E}^{+q2} = e^{+p}h_{p}{}^{q} + e^{-\dot{p}}\chi^{++}_{\dot{p}q} + e^{\pm\pm}\Psi^{+}_{\pm\pm q} \tag{2.27}$$

into (2.26), one finds the expression for $f^{\pm\pm I}$ and the restrictions on the coefficient superfields χ, h, Ψ. Then the integrability conditions for Eqs. (2.27) as well as the Maurer Cartan equations

$$\mathcal{D}f^{++I} \equiv df^{++I} - f^{++I} \wedge \omega + f^{++J} \wedge A^{IJ} = 0, \tag{2.28}$$

$$\mathcal{D}f^{--I} \equiv df^{--I} + f^{--I} \wedge \omega + f^{--J} \wedge A^{IJ} = 0, \tag{2.29}$$

$$d\tilde{\omega} = \frac{1}{2} f^{--I} \wedge f^{++I}, \tag{2.30}$$

$$F^{IJ} = dA^{IJ} + A^{IK} \wedge A^{KJ} = f^{--[I} \wedge f^{++J]} \tag{2.31}$$

shall be investigated [4]. The calculations are simplified essentially due to the theorems about dependence of components of Eqs. (2.26), (2.27), (2.28)–(2.31) (see Appendix in Ref. [23] and refs. therein).

The complete investigation of the equations (2.26), (2.27), (2.28)–(2.31) for the case of flat target $D = 10$ type IIB superspace has been performed in [23]. For our consideration it is essential that, in the frame of the above mentioned assumptions (2.18), (2.19), the general solution of the superembedding equation for the fermionic forms is

$$\hat{E}^{-\dot{q}1} - a\hat{E}^{-\dot{q}2} = \hat{E}^{++} \Psi_{++\dot{q}}^{\;-}, \qquad \hat{E}^{+q2} + a\hat{E}^{+q1} = \hat{E}^{--} \Psi_{--q}^{\;+}, \tag{2.32}$$

where a is a constant parameter $da = 0$. In other words, the following fermionic equations are contained in the list of consequences of the superembedding equation

$$\hat{E}^{++} \wedge (\hat{E}^{-\dot{q}1} - a\hat{E}^{-\dot{q}2}) = 0, \qquad \hat{E}^{--} \wedge (\hat{E}^{+2q} + a\hat{E}^{+1q}) = 0 \tag{2.33}$$

The bosonic equations of motion are specified completely by the fermionic equations and the superembedding condition [23].

To clarify the meaning of the numerical parameter a in the general solution (2.32) we are going to use the generalized action principle and obtain superfield equations of motion for superstring the super-D1-brane.

3 Generalized action and superfield equations of motion for type IIB superstring

3.1 Generalized action

The generalized action for D=10 type IIB superstring is defined as an integral of a Lagrangian form $\hat{\mathcal{L}}_2$ over a bosonic surface $\mathcal{M}^2$ embedded into the worldsheet superspace

[4] Eqs. (2.28), (2.29), (2.30), (2.31) appear as integrability conditions for Eqs. (2.20), (2.19). Note that they give rise to a superfield generalization of the Peterson–Codazzi, Gauss and Ricci equations of the surface theory. In curved superspace Eqs. (2.28)- (2.31) acquire the form (see Appendix B)

$$\mathcal{D}\tilde{f}^{\pm\pm I} = U^{\underline{a}\pm\pm} R_{\underline{a}}{}^{\underline{b}} U_{\underline{b}}^{I}, \qquad d\tilde{\omega} = \frac{1}{2}\tilde{f}^{--I} \wedge \tilde{f}^{++I} + \frac{1}{2} U^{\underline{a}--} R_{\underline{a}}{}^{\underline{b}} U_{\underline{b}}^{++}, \quad \tilde{F}^{IJ} = \tilde{f}^{--[I} \wedge \tilde{f}^{++J]} + U^{\underline{a}I} R_{\underline{a}}{}^{\underline{b}} U_{\underline{b}}^{J}.$$

$$S_{IIB} = \int_{\mathcal{M}^2} \hat{\mathcal{L}}_2 = \int_{\mathcal{M}^2} \left(\frac{1}{2} e^{\frac{1}{2}\Phi(\hat{Z})} \hat{E}^{++} \wedge \hat{E}^{--} - \hat{B}_2 \right). \qquad (3.1)$$

$$\mathcal{M}^2 \in \Sigma^{(2|8+8)}: \qquad \eta^{+q} = \eta^{+q}(\xi^m), \quad \eta^{-\dot{q}} = \eta^{-\dot{q}}(\xi^m). \qquad (3.2)$$

All the field variables entering the Lagrangian form $\hat{\mathcal{L}}_2$ should be considered as superfields, but taken on the bosonic surface (3.2). E.g. the embedding of $\mathcal{M}^2$ into the target superspace is determined by

$$\mathcal{M}^2 \in \underline{\mathcal{M}}^{(10|16+16)}: \quad Z^{\underline{M}} = \hat{Z}^{\underline{M}}(\zeta(\xi)) = \hat{Z}^{\underline{M}}(\xi, \eta(\xi)) \quad \Leftrightarrow \quad \begin{cases} X^{\underline{m}} = \hat{X}^{\underline{m}}(\xi, \eta(\xi)), \\ \Theta^{\underline{\mu}1} = \hat{\Theta}^{\underline{\mu}1}(\xi, \eta(\xi)), \\ \Theta^{\underline{\mu}2} = \hat{\Theta}^{\underline{\mu}2}(\xi, \eta(\xi)). \end{cases} \qquad (3.3)$$

The fermionic fields $\eta^{+q}(\xi^m), \eta^{-\dot{q}}(\xi^m)$ can be regarded as worldsheet Goldstone fermions [40]. As their kinetic term is absent in (3.1), the worldsheet supersymmetry is not broken. However, their presence is important, because it provides the possibility to treat the equations of motion derived from the action (3.1) as *superfield equations* [24] (see also below).

The Lagrangian form $\hat{\mathcal{L}}_2$ of type IIB superstring (3.1) is constructed from the pull–backs of the supervielbein forms of type IIB superspace (1.8), (1.9) and light–like vector Lorentz harmonics $U^{++}_{\underline{m}}(\zeta), U^{--}_{\underline{m}}(\zeta)$ (2.6). The first term is essentially the weight product of two bosonic supervielbeins from the adapted frame (2.7) while the second term contains the pull–back of NS-NS superform B_2 ($\hat{B}_2 = \frac{1}{2} d\hat{Z}^{\underline{M}}(\zeta) \wedge dZ^{\underline{N}}(\zeta) B_{\underline{NM}}(\hat{Z}(\zeta))$) with the flat superspace value

$$B_2 = i\Pi^{\underline{m}} \wedge \left(d\Theta^1 \sigma_{\underline{m}} \Theta^1 - i d\Theta^2 \sigma_{\underline{m}} \Theta^2 \right) + d\Theta^1 \sigma^{\underline{m}} \Theta^1 \wedge d\Theta^2 \sigma_{\underline{m}} \Theta^2. \qquad (3.4)$$

$\hat{\Phi} = \Phi(\hat{Z}(\zeta))$ is the image of the dilaton superfield $\Phi(Z)$ on the worldsheet superspace. It vanishes in the flat type IIB superspace.

Thus, in flat superspace (1.8), (1.9), (3.4) the Lagrangian form of the generalized action (3.1) is

$$\hat{\mathcal{L}}_2 = \frac{1}{2} \hat{E}^{++} \wedge \hat{E}^{--} - i\hat{\Pi}^{\underline{m}} \wedge \left(d\hat{\Theta}^1 \sigma_{\underline{m}} \hat{\Theta}^1 - i d\hat{\Theta}^2 \tilde{\sigma}_{\underline{m}} \hat{\Theta}^2 \right) + d\hat{\Theta}^1 \sigma^{\underline{m}} \hat{\Theta}^1 \wedge d\hat{\Theta}^2 \tilde{\sigma}_{\underline{m}} \hat{\Theta}^2. \quad (3.5)$$

3.2 Variation of generalized action.

The variations of the generalized action (3.1), (3.5) can be derived easily with the use of the seminal formula for the Lee derivative

$$\delta \mathcal{L}_2 = i_\delta d\mathcal{L}_2 + d(i_\delta \mathcal{L}_2) \qquad (3.6)$$

where the symbol i_δ is defined by $i_\delta dZ^{\underline{M}} \equiv \delta \hat{Z}^{\underline{M}}$. The derivatives of the harmonic variables shall be regarded as ones (2.23)–(2.24) which do not break the constraints (2.6). Thus

$$i_\delta dU^{++} \equiv \delta U^{++} = U^I i_\delta f^{++I} + U^{++} i_\delta \omega, \qquad i_\delta dU^{--} \equiv \delta U^{--} = U^I i_\delta f^{--I} + U^{--} i_\delta \omega,$$

$$i_\delta dU^I \equiv \delta U^I = \frac{1}{2} U^{--} i_\delta f^{++I} + \frac{1}{2} U^{++} i_\delta f^{--I}.$$

The contractions of the Cartan forms $i_\delta\omega, i_\delta A^{IJ}, i_\delta f^{\pm\pm I}$ shall be considered as independent variations of the harmonic variables.

It is convenient to replace the basis of 'holonomic variations' $\delta\hat{Z}^{\underline{M}}$ by

$$i_\delta E^{\pm\pm} = \delta\hat{Z}^{\underline{M}} E^{\pm\pm}_{\underline{M}}(\hat{Z}), \qquad i_\delta E^{I} = \delta\hat{Z}^{\underline{M}} E^{I}_{\underline{M}}(\hat{Z}), \tag{3.7}$$

$$i_\delta E^{+q1,2} = \delta\hat{Z}^{\underline{M}} E^{+q1,2}_{\underline{M}}(\hat{Z}), \qquad i_\delta E^{-\dot{q}1,2} = \delta\hat{Z}^{\underline{M}} E^{-\dot{q}1,2}_{\underline{M}}(\hat{Z}).$$

In the flat superspace (1.8), (1.9) the basis (3.7) becomes

$$i_\delta E^{\pm\pm} \equiv i_\delta \Pi^{\underline{m}} U^{\pm\pm}_{\underline{m}}, \qquad i_\delta E^{I} \equiv i_\delta \Pi^{\underline{m}} U^{I}_{\underline{m}}, \tag{3.8}$$

$$i_\delta E^{-\dot{q}1} = \delta\Theta^{\underline{\mu}1} V_{\underline{\mu}}^{-\dot{q}}, \qquad i_\delta E^{+q2} = \delta\Theta^{\underline{\mu}2} V_{\underline{\mu}}^{+q},$$

where

$$i_\delta \Pi^{\underline{m}} \equiv \delta X^{\underline{m}} - i\delta\Theta^{1\underline{\mu}} \sigma^{\underline{m}}_{\underline{\mu\nu}} \Theta^{1\underline{\nu}} - i\delta\Theta^{2\underline{\mu}} \sigma^{\underline{m}}_{\underline{\mu\nu}} \Theta^{2\underline{\nu}}.$$

3.3 Superfield equations of motion and gauge symmetries.

With the above notations (2.14), (2.15), (2.20) the equations of motion which follows from the generalized action (3.1) –(3.3) in the flat target superspace are

$$\hat{E}^{I} \equiv \hat{\Pi}^{\underline{m}} U^{I}_{\underline{m}} = 0, \tag{3.9}$$

$$\hat{E}^{++} \wedge \hat{E}^{-\dot{q}1} \equiv \hat{\Pi}^{\underline{m}} \wedge d\hat{\Theta}^{1\underline{\mu}} V_{\underline{\mu}\dot{q}}^{\ -} U^{++}_{\underline{m}} = 0, \tag{3.10}$$

$$\hat{E}^{--} \wedge \hat{E}^{+2q} \equiv \hat{\Pi}^{\underline{m}} \wedge d\hat{\Theta}^{2\underline{\mu}} V_{\underline{\mu}q}^{\ +} U^{--}_{\underline{m}} = 0. \tag{3.11}$$

$$\hat{M}_2^I \equiv \hat{E}^{--} \wedge f^{++I} - \hat{E}^{++} \wedge f^{--I} - 4i\left(\hat{E}^{+q1} \wedge \hat{E}^{-\dot{q}1} - \hat{E}^{+2q} \wedge \hat{E}^{-2\dot{q}}\right)\gamma^I_{q\dot{q}} = 0. \tag{3.12}$$

The variations of the Goldstone fermions $\eta^{+q}(\xi^m), \eta^{-\dot{q}}(\xi^m)$ do not produce independent equations [24]. Thus Eqs. (3.9)–(3.12) are satisfied on *any* bosonic surface $\mathcal{M}^2$ (3.2) in the worldsheet superspace $\Sigma^{(2|16)}$ (1.2). As the set of all such bosonic surfaces covers the whole superspace $\Sigma^{(2|16)}$, one concludes that Eqs. (3.9)- (3.12) can be treated as *superfield equations for the differential forms defined on the whole worldsheet superspace* [24]. In this case all the field variables in (3.9)-(3.12) shall be treated as worldsheet superfields $\hat{X} = \hat{X}(\zeta) = \hat{X}(\xi,\eta)$, $\hat{\Theta}^{1,2} = \hat{\Theta}^{1,2}(\zeta) = \hat{\Theta}^{1,2}(\xi,\eta)$, $U = U(\zeta) = U(\xi,\eta)$ without restrictions to a bosonic surface. Thus the generalized action principle [24, 26] can be used as a dynamical basis for the superembedding approach [23, 24, 25, 28].

For our study it is especially important that Eqs. (3.10), (3.11) coincides with (2.33) taken with $a = 0$.

When the fermionic worldsheet coordinates (Goldstone fields) vanish $\eta = 0$, the set of equations (3.9)–(3.12) becomes equivalent to the standard equations of motion for the type IIB Green-Schwarz superstring [37]. In this case the basic fermionic forms can be decomposed on the bosonic ones

$$\mathcal{M}^2_0: \qquad E^{+q2} = E^{++}E^{+q2}_{++} + E^{--}E^{+q2}_{--}, \qquad E^{-\dot{q}1} = E^{++}E^{-\dot{q}1}_{++} + E^{--}E^{-\dot{q}1}_{--},$$

and one can write Eqs. (3.10) in the form (1.11) with $e^{\pm\pm} = E^{\pm\pm}$.

The external derivative of the Lagrangian form can be used for derivation of Eqs. (3.9)–(3.12) and contains as well an information about local (gauge) symmetries of the superstring. In flat target superspace it is

$$d\hat{\mathcal{L}}_2^{IIB} = -2i\hat{E}^{++}\wedge\hat{E}^{-\dot{q}1}\wedge\hat{E}^{-\dot{q}1} + 2i\hat{E}^{++}\wedge\hat{E}^{+q2}\wedge\hat{E}^{+q2} + \frac{1}{2}\hat{E}^I\wedge\hat{M}_2^I + \propto \hat{E}^I\wedge\hat{E}^J, \tag{3.13}$$

where $\hat{M}_2^I$ is defined in Eq. (3.12).

Eqs. (3.9)-(3.12) appear as a result of the variations with respect to $i_\delta f^{\pm\pm I}$, $i_\delta E^{-\dot{q}1} = \delta\Theta^{\underline{\mu}1}V_{\underline{\mu}}^{-\dot{q}}$, $i_\delta E^{+q2} = \delta\Theta^{\underline{\mu}2}V_{\underline{\mu}}^{+q}$ and $i_\delta E^I \equiv i_\delta\Pi^{\underline{m}}U_{\underline{m}}^I$. The remaining (super)field variations do not produce any nontrivial equations and, thus, can be identified with the parameters of *local (gauge) symmetries of the model*. Evident gauge symmetries of the action (3.1) are $SO(1,1)\times SO(8)$ ($i_\delta\omega,\ i_\delta A^{IJ}$) and reparametrization ($i_\delta E^{\pm\pm}$). The parameters of the *stringy κ–symmetry* can be identified with the contractions of those fermionic forms which are absent in the first line of Eq. (3.13)

$$\kappa^{+q} \equiv i_\delta\hat{E}^{+q1} = \delta\hat{\Theta}^{1\underline{\mu}}V_{\underline{\mu}}^{+q}, \qquad \kappa^{-\dot{q}} \equiv i_\delta\hat{E}^{-2\dot{q}} = \delta\hat{\Theta}^{2\underline{\mu}}V_{\underline{\mu}}^{-\dot{q}}. \tag{3.14}$$

The second equalities of Eqs. (3.14) are vailed for the flat target superspace only. In this case the κ–symmetry transformations of the generalized action are defined by

$$\delta_\kappa\hat{\Theta}^{\underline{\mu}1}(\zeta(\xi)) = \epsilon^{+q}(\xi)V_q^{-\underline{\mu}}(\zeta(\xi)), \qquad \delta_\kappa\hat{\Theta}^{\underline{\mu}2}(\zeta(\xi)) = \epsilon^{-\dot{q}}(\xi)V_{\dot{q}}^{+\underline{\mu}}(\zeta(\xi)). \tag{3.15}$$

$$i_\kappa\Pi^{\underline{m}} = 0 \quad \Leftrightarrow \quad \delta_\kappa X^{\underline{m}} = i\delta_\kappa\Theta^{1\underline{\mu}}\sigma^{\underline{m}}_{\underline{\mu}\underline{\nu}}\Theta^{1\underline{\nu}} + i\delta_\kappa\Theta^{2\underline{\mu}}\sigma^{\underline{m}}_{\underline{\mu}\underline{\nu}}\Theta^{2\underline{\nu}}$$

and by the quite complicated transformations of harmonic variables

$$\delta_\kappa U_{\underline{m}}^{\pm\pm} = U_{\underline{m}}^I i_\kappa f^{\pm\pm I}, \qquad \delta_\kappa U_{\underline{m}}^I = \frac{1}{2}U_{\underline{m}}^{++}i_\kappa f^{--I} + \frac{1}{2}U_{\underline{m}}^{--}i_\kappa f^{++I} \tag{3.16}$$

whose explicit form is unessential for our consideration[5].

In accordance with [15, 28], the worldsheet supersymmetry has to be in one–to–one correspondence with the κ–symmetry. Thus the parameters of the worldsheet supersymmetry ϵ should have the same $SO(1,1)\times SO(8)$ 'quantum numbers' as the parameters of the irreducible κ–symmetry. Moreover, on the bosonic surface $\mathcal{M}^2$ (3.3) one should arrive at

$$\epsilon^{+q} = \kappa^{+q} + \ldots, \qquad \epsilon^{-\dot{q}} = \kappa^{-\dot{q}} + \ldots. \tag{3.17}$$

On the other hand, the parameters of the local worldsheet supersymmetry can be identified with contractions of the worldsheet fermionic supervielbein forms. Thus the 'quantum numbers' ($SO(1,1)$ weight and $SO(8)$ representation) of the fermionic worldsheet supervielbein are fixed by κ–symmetry as it is indicated in (1.6)

$$\epsilon^{+q} = i_\delta e^{+q} = \delta\zeta^M e_M^{+q} \qquad \epsilon^{-\dot{q}} = i_\delta e^{-\dot{q}} = \delta\zeta^M e_M^{-\dot{q}}.$$

Moreover, the local supersymmetry transformations of the fermionic supervielbein

$$\delta e^{+q} = Di_\delta e^{+q} + i_\delta(De^{+q}) = D\epsilon^{+q} + \ldots, \qquad \delta e^{-\dot{q}} = Di_\delta e^{-\dot{q}} + i_\delta(De^{-\dot{q}}) = D\epsilon^{-\dot{q}} + \ldots \tag{3.18}$$

[5] To find $\delta_\kappa U^{\pm\pm}, \delta_\kappa U^I$, one has to solve equation $i_\kappa(d\mathcal{L}_2) = 0$ with respect to $i_\kappa f^{--I}$, $i_\kappa f^{++I}$ under the suppositions (3.15), $i_\kappa\omega = 0, i_\kappa A^{IJ} = 0$.

basically coincide with the κ–symmetry transformations of the induced supervielbein forms (2.18) *lifted to the whole worldsheet superspace*

$$\delta_\kappa e^{+q} = \delta_\kappa (d\Theta^{\underline{\mu}1} V_{\underline{\mu}q}^{\;+}) = D\kappa^{+q} + \ldots, \qquad \delta_\kappa e^{-\dot{q}} = \delta_\kappa (d\Theta^{\underline{\mu}2} V_{\underline{\mu}\dot{q}}^{\;-}) = D\kappa^{-\dot{q}} + \ldots \tag{3.19}$$

Thus the choice of proper fermionic supervielbein forms (1.6) is motivated by the *irreducible* κ–symmetry of the generalized action of type IIB superstring.

The main conclusions of our consideration in the present section are as follows:

- the superfield fermionic equations of motion (3.10) for type IIB superstring coincide with the general solution of the superembedding equations (2.33) taken with $a = 0$.
- the parameters of stringy κ–symmetry can be identified as in Eq. (3.14). This indicates that a half of Θ^1 and a half of Θ^2 (but not Θ^1 or Θ^2 completely) can be gauged away by the κ-symmetry in a component approach. In the superfield approach one can instead fix the gauge $\Theta^{+q1} \equiv \Theta^{\underline{\mu}1} V_{\underline{\mu}q}^{\;+} = \eta^{+q}$, $\Theta^{-\dot{q}2} \equiv \Theta^{\underline{\mu}2} V_{\underline{\mu}\dot{q}}^{\;-} = \eta^{-\dot{q}}$.
- The κ–symmetry determines the proper choice of the fermionic supervielbeine forms of the worldsheet superspace. In particular the forms induced by the embedding in accordance with Eqs. (2.18) are appropriate.

4 Super–D1–brane. Generalized action, superfield equations and gauge symmetries

The generalized action for super-D1-brane (Dirichlet superstring) has the form

$$S_{D1} = \int_{\mathcal{M}^{1+1}} \hat{\mathcal{L}}_2^{D1} = \int_{\mathcal{M}^{1+1}} \frac{1}{2} e^{-\frac{1}{2}\hat{\Phi}} \hat{E}^{++} \wedge \hat{E}^{--} \sqrt{1 - (F^{(0)})^2} + \tag{4.1}$$

$$+ \int_{\mathcal{M}^{1+1}} \left[Q_0 \left(e^{-\frac{1}{2}\Phi} \left(dA - \hat{B}_2 \right) - \frac{1}{2} \hat{E}^{++} \wedge \hat{E}^{--} F^{(0)} \right) + \hat{C}_2 + \hat{C}_0 (dA - \hat{B}_2) \right],$$

where, for simplicity, we put tension equal to unity $T = 1$ (as we did in Eq. (3.1) for the fundamental superstring). In (4.1) $\hat{C}_0 \equiv C_0 \left(\hat{Z}(\xi, \eta(\xi)) \right)$ is the axion superfield restricted to the surface $\mathcal{M}^2$, $\hat{C}_2$ is the pull-back of the RR superform and $Q_0 = Q_0(\xi, \eta(\xi))$ is a Lagrange multiplier superfield. The remaining notations are the same as in Section 3.

Eq. (4.1) can be obtained from the general formula of Ref. [26] by substitution $F_{ab} = \epsilon_{ab} F^{(0)}$ for the two–dimensional auxiliary tensor superfield. Then the Born-Infeld square root acquires the form $\sqrt{-det(\eta_{ab} + F_{ab})} = \sqrt{1 - (F^{(0)})^2}$, which indicates that the scalar superfield $F^{(0)}$ lives in the interval $-1 < F^{(0)} < 1$.

Varying the generalized action (4.1) with respect to the auxiliary field $F^{(0)}$ one finds the expression

$$Q_0 = -e^{-\frac{1}{2}\hat{\Phi}} \frac{F^{(0)}}{\sqrt{1 - (F^{(0)})^2}}, \tag{4.2}$$

for the Lagrange multiplier Q_0. The variation δQ_0 produces the superfield constraint for the 2-dimensional worldsheet gauge superfield

$$e^{-\frac{1}{2}\hat{\Phi}}\left(dA - \hat{B}_2\right) = \frac{1}{2}\hat{E}^{++}\wedge\hat{E}^{--}F^{(0)}, \tag{4.3}$$

while the variation of (*a priori* unconstrained) gauge superfield $A = d\zeta^M(\xi)A_M(\zeta(\xi))$ results in the 2-dimensional Born-Infeld equation

$$d\left(Q_0 e^{-\frac{1}{2}\hat{\Phi}} + \hat{C}_0\right) = 0. \tag{4.4}$$

In flat superspace (1.8), (1.9), where $C_0 = 0$, $\Phi = 0$ and

$$C_2 = i\Pi^{\underline{m}}\wedge\left(d\Theta^2\sigma_{\underline{m}}\Theta^1 - i\Theta^2\sigma_{\underline{m}}d\Theta^1\right) + (d\Theta^1\sigma^{\underline{m}}d\Theta^1 - d\Theta^2\sigma^{\underline{m}}d\Theta^2)\wedge\Theta^2\sigma_{\underline{m}}\Theta^1, \tag{4.5}$$

Eq. (4.4) reduces to

$$dQ_0 = 0 \qquad \Rightarrow \qquad dF^{(0)} = 0. \tag{4.6}$$

and implies that the field strength $F^{(0)}$ of the worldsheet gauge field is a constant.

The variations of the harmonics ($\delta U^{\pm\pm} = U^I i_\delta f^{\pm\pm I}$) result in the *same superembedding equation as for the case of fundamental superstring*

$$\hat{E}^I \equiv \hat{E}^{\underline{a}}U^I_{\underline{a}} = 0. \tag{4.7}$$

If one considers the derivative of the Lagrangian form taken on the surface of algebraic and geometric equations (4.2), (4.3), (4.7) $d\mathcal{L}_2^{D1}|_s$, after some tedious but straightforward algebraic manipulations one finds that it contains only 16 of 32 independent fermionic forms $\hat{E}^{2+}_q, \hat{E}^{1+}_q, \hat{E}^{2-}_{\dot{q}}, \hat{E}^{1-}_{\dot{q}}$ (see Appendix B)

$$\left(\hat{E}^{2-}_{\dot{q}} - \sqrt{\frac{1-F^{(0)}}{1+F^{(0)}}}\hat{E}^{1-}_{\dot{q}}\right), \qquad \left(\hat{E}^{2+}_q + \sqrt{\frac{1+F^{(0)}}{1-F^{(0)}}}\hat{E}^{1+}_q\right). \tag{4.8}$$

This is the reflection of the fermionic κ-symmetry on the level of Noether identities. The parameters of the κ-symmetry can be identified with the contractions of some linear combinations of the forms which are independent on (4.8). In particular we can define

$$\tilde{\kappa}^{1+q} = i_\delta\hat{E}^{1+}_q, \quad \tilde{\kappa}^{2-}_{\dot{q}} = i_\delta\hat{E}^{1-}_{\dot{q}}, \quad \Leftrightarrow \quad \tilde{\kappa}^{\underline{\alpha}} = \delta\hat{Z}^{\underline{M}}\hat{E}^{\underline{\alpha}1}_{\underline{M}}(\hat{Z}), \tag{4.9}$$

In the case of flat target superspace Eq. (4.9) becomes $\tilde{\kappa}^{\underline{\mu}} = \delta\hat{\Theta}^{\underline{\mu}1}$ and means that the Grassmann coordinate field $\hat{\Theta}^{\underline{\mu}1}$ can be gauged away by κ–symmetry in a component approach or identified with Grassmann coordinate of the worldsheet superspace $\Theta^{\underline{\mu}1} = \eta^{\underline{\mu}}$ in the superfield approach.

On the other hand, *the parameter of the irreducible κ–symmetry can be defined in the same way as in the case of type IIB superstring* (3.14)

$$\kappa^{+q} \equiv i_\delta\hat{E}^{+q1} = \delta\hat{\Theta}^{1\underline{\mu}}V^{+q}_{\underline{\mu}}, \qquad \kappa^{-\dot{q}} \equiv i_\delta\hat{E}^{-2\dot{q}} = \delta\hat{\Theta}^{2\underline{\mu}}V^{-\dot{q}}_{\underline{\mu}}.$$

Thus the worldsheet superspace with the geometry induced by embedding in accordance with Eqs. (2.17), (2.18), (2.19) is proper for the description of both the superstring and super-D1-brane.

5 Universal description of superstring and super-D1-brane by superembedding. Flat type IIB superspace.

In flat type IIB superspace the superfield generalization of the fermionic equations of motion of super-D1-brane

$$\hat{E}^{++} \wedge \left(\hat{E}^{2-}_{\dot{q}} - \sqrt{\frac{1-F^{(0)}}{1+F^{(0)}}} \hat{E}^{1-}_{\dot{q}} \right) = 0, \qquad \hat{E}^{--} \wedge \left(\hat{E}^{2+}_{q} + \sqrt{\frac{1+F^{(0)}}{1-F^{(0)}}} \hat{E}^{1+}_{q} \right) = 0, \quad (5.1)$$

can be obtained as a result of variations $i_\delta \hat{E}^{2-}_{\dot{q}}$ and $i_\delta \hat{E}^{2+}_{q}$. The independent part of the superfield generalization of the bosonic coordinate equations of motion appear as a result of the variation with respect to $i_\delta \hat{E}^I = i_\delta \hat{\Pi}^{\underline{m}} U^I_{\underline{m}}$ and are

$$M^I_{2(D1)} \equiv \hat{E}^{--} \wedge f^{++I} - \hat{E}^{++} \wedge f^{--I} - 2i\gamma^I_{q\dot{q}} F^{(0)} \left(\hat{E}^{2+}_{q} \wedge \hat{E}^{2-}_{\dot{q}} + \hat{E}^{1+}_{q} \wedge \hat{E}^{1-}_{\dot{q}} \right) - \quad (5.2)$$
$$-2i\gamma^I_{q\dot{q}} \sqrt{1-(F^{(0)})^2} \left(\hat{E}^{2+}_{q} \wedge \hat{E}^{1-}_{\dot{q}} + \hat{E}^{1+}_{q} \wedge \hat{E}^{2-}_{\dot{q}} \right) = 0.$$

Eq. (5.2) can be obtained as a consequence of the superembedding equation (4.7) and the superfield fermionic equations (5.1) with constant $F^{(0)}$.

Now it is evident that the superfield fermionic equations of motion for the super-D1-brane (5.1) coincide with the general solution (2.33) (or (2.32)) of the superembedding equation (3.9) for

$$a = \sqrt{\frac{1+F^{(0)}}{1-F^{(0)}}}, \qquad da = 0. \quad (5.3)$$

Note that a is a constant due to the 2-dimensional Born-Infeld equation (4.6).

Actually, the scale of the parameter a is unessential and the value of the field strength of the gauge field is in one–to–one correspondence with the value of an angle α which can be regarded as a parameter of the $SO(2)$ symmetry (1.12) of the superembedding equation (1.7)

$$Cos\alpha = \sqrt{\frac{1+F^{(0)}}{2}}. \quad (5.4)$$

which is the flat superspace image of the classical S-duality group $SL(2,R)$. Thus we find that this $SO(2)$ symmetry mix the fundamental superstring with super-D1-brane with all possible values of constant field strength of the gauge field.

More precisely, the map (5.4) corresponds to $SO(2)$ transformations with the parameter $\alpha \in [-\frac{\pi}{2}, \frac{\pi}{2}]$, while the range $\alpha \in [\frac{\pi}{2}, \frac{3\pi}{2}]$, which corresponds to the solution of superembedding equation with negative a, relates superstring with anti–super–D1–brane whose action has opposite sign of the Wess–Zumino term with respect to (4.1) $S_{\bar{D}1} = S_{D1}|_{C_2 \rightarrow -C_2,\ C_0 \rightarrow -C_0}$. Thus we arrive at Eqs. (1.13), (1.14). From the point of view of superembedding approach to super-D1-brane, the fundamental superstring and anti–superstring (with opposite NSNS charge $S_{\bar{I}\bar{I}\bar{B}} = S_{IIB}|_{B_2 \rightarrow -B_2}$) correspond to the non-proper limits for the value of the on–shell gauge field strength $F^{(0)} = -1$ and $F^{(0)} = +1$.

Hence we conclude that the superembedding equation (4.7), (or, equivalently, (1.4)) is S-duality invariant and provides a universal description of the fundamental type IIB superstring and super-D1-brane.

6 Universal description of superstring and super-D1-brane in general supergravity background.

6.1 Constraints and Weyl transformations

In general type IIB supergravity background the supervielbein

$$E^{\underline{A}} = \left(E^{\underline{a}}, E^{\underline{\alpha}1}, E^{\underline{\alpha}2}\right) = dZ^{\underline{M}} E_{\underline{M}}^{\underline{A}}(Z^{\underline{M}}), \tag{6.1}$$

is subject to the basic torsion constraints [41]

$$T^{\underline{a}} \equiv \mathcal{D}E^{\underline{a}} \equiv dE^{\underline{a}} - E^{\underline{b}} \wedge w_{\underline{b}}{}^{\underline{a}} = -i(E^{\underline{\alpha}1} \wedge E^{\underline{\beta}1} + E^{\underline{\alpha}2} \wedge E^{\underline{\beta}2})\sigma^{\underline{a}}_{\underline{\alpha\beta}} \tag{6.2}$$

where

$$\mathcal{D} = E^{\underline{a}}\mathcal{D}_{\underline{a}} + E^{\underline{\alpha}1}\mathcal{D}_{\underline{\alpha}1} + E^{\underline{\alpha}2}\mathcal{D}_{\underline{\alpha}2}$$

is covariant external derivative and

$$w_{\underline{b}}{}^{\underline{a}} = E^{\underline{c}} w_{\underline{cb}}{}^{\underline{a}} + E^{\underline{\alpha}1} w_{\underline{\alpha}1\underline{b}}{}^{\underline{a}} + E^{\underline{\alpha}2} w_{\underline{\alpha}2\underline{b}}{}^{\underline{a}}, \qquad w^{\underline{ab}} = -w^{\underline{ab}} \tag{6.3}$$

is the $SO(1,9)$ spin connection.

The basic constraint (6.2) is invariant under the local Weyl transformations

$$E^{\underline{a}} \rightarrow E^{\underline{a}\prime} = e^{2W} E^{\underline{a}}, \tag{6.4}$$

$$E^{\underline{\alpha}1} \rightarrow E^{\underline{\alpha}1\prime} = e^{W}\left(E^{\underline{\alpha}1} - iE^{\underline{a}}\tilde{\sigma}_{\underline{a}}^{\underline{\alpha\beta}}\nabla_{\underline{\beta}1}W\right), \tag{6.5}$$

$$E^{\underline{\alpha}2} \rightarrow E^{\underline{\alpha}1\prime} = e^{W}\left(E^{\underline{\alpha}2} - iE^{\underline{a}}\tilde{\sigma}_{\underline{a}}^{\underline{\alpha\beta}}\nabla_{\underline{\beta}2}W\right), \tag{6.6}$$

$$\begin{aligned} w^{\underline{ab}} \rightarrow (w^{\underline{ab}})' = w^{\underline{ab}} + \frac{1}{2}E^{[\underline{a}}\nabla^{\underline{b}]}W + 4(\sigma^{\underline{ab}})_{\underline{\alpha}}{}^{\underline{\beta}}\left(E^{\underline{\alpha}1}\nabla_{\underline{\beta}1}W + E^{\underline{\alpha}2}\nabla_{\underline{\beta}2}W\right) + \\ +iE_{\underline{c}}(\tilde{\sigma}^{\underline{abc}})^{\underline{\alpha\beta}}\left(\nabla_{\underline{\alpha}1}W\nabla_{\underline{\beta}1}W + \nabla_{\underline{\alpha}2}W\nabla_{\underline{\beta}2}W\right) \end{aligned} \tag{6.7}$$

with arbitrary superfield parameter $W = W(Z)$. It is evidently invariant as well under the local $SO(2)$ rotations of fermionic supervielbein forms.

When superbranes in supergravity background are considered, it is convenient to introduce a superfield generalizations for all tensor gauge fields involved into the supergravity supermultiplet. Using the string terminology, one can introduce [41, 42] the NSNS 2-form $B_2 = \frac{1}{2}dZ^{\underline{M}} \wedge dZ^{\underline{N}} B_{\underline{NM}}(Z)$, RR 2-form and 4-form $C_2 = \frac{1}{2}dZ^{\underline{M}} \wedge dZ^{\underline{N}} C_{\underline{NM}}(Z)$, $C_4 = \frac{1}{4!}dZ^{\underline{M}_4} \wedge \ldots \wedge dZ^{\underline{M}_1} C_{\underline{M}_1 \ldots \underline{M}_4}(Z)$, as well as axion superfield $C_0(Z)$. The requirement of the κ–symmetry of the super–Dp–brane actions specifies the constraints on the NS-NS and RR field strengths [42]

$$\begin{aligned} H_3 \equiv dB_2 = -ie^{\frac{1}{2}\Phi}E^{\underline{a}} \wedge (E^{\underline{\alpha}1} \wedge E^{\underline{\alpha}1} - E^{\underline{\alpha}2} \wedge E^{\underline{\alpha}2})\sigma_{\underline{a}\ \underline{\alpha\beta}} + \\ +\frac{1}{4}e^{\frac{1}{2}\Phi}E^{\underline{b}} \wedge E^{\underline{a}} \wedge \left(E^{\underline{\alpha}1}\nabla_{\underline{\beta}1}\Phi - E^{\underline{\alpha}2}\nabla_{\underline{\beta}2}\Phi\right)(\sigma_{\underline{ab}})_{\underline{\alpha}}{}^{\underline{\beta}} + \frac{1}{3!}E^{\underline{c}} \wedge E^{\underline{b}} \wedge E^{\underline{a}} H_{\underline{abc}}(Z), \end{aligned} \tag{6.8}$$

$$R_1 = dC_0 = e^{-\Phi}E^{\underline{\alpha}1}\nabla_{\underline{\alpha}2}\Phi - e^{-\Phi}E^{\underline{\alpha}2}\nabla_{\underline{\alpha}1}\Phi + E^{\underline{a}}R_{\underline{a}}, \tag{6.9}$$

$$R_3 = dC_2 - C_0 dB_2 = 2ie^{-\frac{1}{2}\Phi} E^{\underline{a}} \wedge E^{\underline{\alpha}2} \wedge E^{\underline{\beta}1} \sigma_{\underline{a}\underline{\alpha}\underline{\beta}} + \tag{6.10}$$

$$+\frac{1}{4} e^{-\frac{1}{2}\Phi} E^{\underline{b}} \wedge E^{\underline{a}} \wedge \left(E^{\underline{\alpha}1} \nabla_{\underline{\alpha}2} \Phi + E^{\underline{\alpha}2} \nabla_{\underline{\alpha}1} \Phi \right) (\sigma_{\underline{ab}})_{\underline{\alpha}}{}^{\underline{\beta}} + \frac{1}{3!} E^{\underline{c}} \wedge E^{\underline{b}} \wedge E^{\underline{a}} R_{\underline{abc}}(Z),$$

$$R_5 = dC_4 - C_2 \wedge H_3 = 2i \frac{1}{3!} E^{\underline{c}} \wedge E^{\underline{b}} \wedge E^{\underline{a}} \wedge E^{\underline{\alpha}2} \wedge E^{\underline{\beta}1} \sigma_{\underline{abc}\ \underline{\alpha\beta}} + \ldots \tag{6.11}$$

Actually, the constraints on the superforms of type IIB supergravity can be written in a manifestly $SL(2,R)$ invariant manner [43]. The lack of manifest $SL(2,R)$ invariance for the constraints (6.8) – (6.11) obtained from the consideration of super–Dp–brane actions reflects the fact that not all the super–Dp–branes are invariant under the S–duality.

6.2 Superfield equations in general supergravity background

In general supergravity background the generalized actions for superstring and super–D1–brane both produce the superembedding equation

$$\hat{E}^I \equiv \hat{E}^{\underline{a}} U_{\underline{a}}^I = 0. \tag{6.12}$$

The superfield fermionic equations for type IIB superstring have the form

$$\hat{E}^{++} \wedge \left(\hat{E}^{-\dot{q}1} - \frac{i}{8} \hat{E}^{--} (V\nabla_1)^{+}_{\dot{q}} \Phi \right) = 0, \tag{6.13}$$

$$\hat{E}^{--} \wedge \left(\hat{E}^{2+}_{q} - \frac{i}{8} \hat{E}^{++} (V\nabla_2)^{-}_{q} \Phi \right) = 0, \tag{6.14}$$

while for the super-D1–brane they are

$$\hat{E}^{++} \wedge \left(\hat{E}^{2-}_{\dot{q}} + \sqrt{\frac{1-F^{(0)}}{1+F^{(0)}}} \hat{E}^{1-}_{\dot{q}} \right) - \tag{6.15}$$

$$-\frac{i}{4} \hat{E}^{++} \wedge \hat{E}^{--}\ V^{+\underline{\alpha}}_{\dot{q}} \left((F - \frac{1}{2}) \nabla_{\underline{\alpha}2} \Phi - \sqrt{\frac{1-F^{(0)}}{1+F^{(0)}}} (F + \frac{1}{2}) \nabla_{\underline{\alpha}1} \Phi \right) = 0,$$

$$\hat{E}^{--} \wedge \left(\hat{E}^{2+}_{q} - \sqrt{\frac{1+F^{(0)}}{1-F^{(0)}}} \hat{E}^{1+}_{q} \right) + \tag{6.16}$$

$$+\frac{i}{4} \hat{E}^{--} \wedge \hat{E}^{++}\ V^{-\underline{\alpha}}_{q} \left((F + \frac{1}{2}) \nabla_{\underline{\alpha}2} \Phi + \sqrt{\frac{1+F^{(0)}}{1-F^{(0)}}} (F - \frac{1}{2}) \nabla_{\underline{\alpha}1} \Phi \right) = 0.$$

The complete form of the bosonic coordinate equations for type IIB superstring ($M^I_{2(IIB)} = 0$) and super–D1-brane ($M^I_{2(D1)} = 0$) in general supergravity background can be found in Appendix B.

The Born–Infeld equation for super–D1–brane (4.4), (4.2) involves inputs from axion and dilaton. The general solution expresses the field strength $F^{(0)}$ through a constant c and images of axion and dilaton superfields on the worldsheet superspace

$$F^{(0)}(\zeta) = \frac{(c+\hat{C}_0)e^{\hat{\Phi}}}{\sqrt{1+(c+\hat{C}_0)^2 e^{2\hat{\Phi}}}}, \qquad \hat{C}_0 \equiv C_0\left(\hat{Z}(\zeta)\right), \qquad \hat{\Phi} \equiv \Phi\left(\hat{Z}(\zeta)\right), \qquad c = const. \tag{6.17}$$

The superembedding equation (6.12) and the fermionic equations of motion (6.13), (6.14), or (6.15), (6.16) together with the Born–Infeld equation, specify bosonic coordinate equations completely as they do in the case of flat superspace. Thus to study the relation between super–D1–brane and superstring it is enough to investigate the relation between the fermionic equations (6.13), (6.14) and (6.15), (6.16).

6.3 Super–Weyl transformations of fermionic equations

Let us begin with an instructive observation about superstring fermionic equations (6.13), (6.14). They can be written in an equivalent form

$$\hat{E}^{++} \wedge \left(\hat{E}^{\underline{\alpha}1} - \frac{i}{8}\hat{E}^{\underline{a}}\tilde{\sigma}_{\underline{a}}^{\underline{\alpha\beta}}\nabla_{\underline{\beta}1}\Phi\right) V_{\underline{\alpha}\dot{q}}^{\ -} = 0, \tag{6.18}$$

$$\hat{E}^{--} \wedge \left(\hat{E}^{\underline{\alpha}2} - \frac{i}{8}\hat{E}^{\underline{a}}\tilde{\sigma}_{\underline{a}}^{\underline{\alpha\beta}}\nabla_{\underline{\beta}2}\Phi\right) V_{\underline{\alpha}q}^{\ +} = 0, \tag{6.19}$$

where it becomes evident that these equations can be reduced to (3.10), (3.11) by super–Weyl transformations (6.4)–(6.7) with the superfield parameter $W = \frac{1}{8}\Phi(Z)$.

To find a proper super–Weyl transform for simplification of the super–D1–brane equations, it is convenient to introduce a 10–dimensional composed scalar superfield

$$F(Z) = \frac{(c+C(Z))e^{\Phi(Z)}}{\sqrt{1+(c+C(Z))^2 e^{2\Phi(Z)}}}, \tag{6.20}$$

which can be used to write the solution (6.17) of the Born–Infeld equation (4.4), (4.2) in the form

$$F^{(0)}(\zeta) = F\left(\hat{Z}(\zeta)\right), \tag{6.21}$$

In accordance with the constraints of type IIB supergravity (6.9), the spinor covariant derivatives of the axion C_0 superfield are expressed through the ones of the dilaton superfield Φ

$$\nabla_{\underline{\alpha}1}C_0 = e^{-\Phi}\nabla_{\underline{\alpha}2}\Phi, \qquad \nabla_{\underline{\alpha}2}C_0 = -e^{-\Phi}\nabla_{\underline{\alpha}1}\Phi. \tag{6.22}$$

Hence, the spinor covariant derivatives of the superfield F (6.20) have the form of $SO(2)$ transformed Grassmann derivatives of dilaton superfield times a local scale

$$\frac{1}{1-F^2}\nabla_{\underline{\alpha}1}F = F\nabla_{\underline{\alpha}1}\Phi + \sqrt{1-F^2}\nabla_{\underline{\alpha}2}\Phi, \tag{6.23}$$

$$\frac{1}{1-F^2}\nabla_{\underline{\alpha}2}F = -\sqrt{1-F^2}\nabla_{\underline{\alpha}1}\Phi + F\nabla_{\underline{\alpha}2}\Phi.$$

Taking into account Eqs. (6.23) and performing some straightforward calculations one can find that the super–Weyl transformation (6.4)–(6.7) with the parameter

$$e^W = \left(e^{-2\Phi} + (c+C_0)^2\right)^{\frac{1}{8}} \tag{6.24}$$

(supplemented with a trivial rescaling of the equations) can be used to recast Eqs. (6.15), (6.16) to

$$\hat{E}^{++} \wedge \Big(\sqrt{\frac{1-F^{(0)}}{2}} \Big(\hat{E}^{\underline{\alpha}1} - \frac{i}{8} \hat{E}^{\underline{a}} \tilde{\sigma}_{\underline{a}}^{\underline{\alpha\beta}} \nabla_{\underline{\beta}1} \Phi \Big) + \tag{6.25}$$

$$+ \sqrt{\frac{1+F^{(0)}}{2}} \Big(\hat{E}^{\underline{\alpha}2} - \frac{i}{8} \hat{E}^{\underline{a}} \tilde{\sigma}_{\underline{a}}^{\underline{\alpha\beta}} \nabla_{\underline{\beta}2} \Phi \Big) \Big) V_{\underline{\alpha}\dot{q}}^{\ -} = 0,$$

$$\hat{E}^{--} \wedge \Big(- \sqrt{\frac{1+F^{(0)}}{2}} \Big(\hat{E}^{\underline{\alpha}1} - \frac{i}{8} \hat{E}^{\underline{a}} \tilde{\sigma}_{\underline{a}}^{\underline{\alpha\beta}} \nabla_{\underline{\beta}1} \Phi \Big) + \tag{6.26}$$

$$+ \sqrt{\frac{1-F^{(0)}}{2}} \Big(\hat{E}^{\underline{\alpha}2} - \frac{i}{8} \hat{E}^{\underline{a}} \tilde{\sigma}_{\underline{a}}^{\underline{\alpha\beta}} \nabla_{\underline{\beta}2} \Phi \Big) \Big) V_{\underline{\alpha}q}^{\ +} = 0.$$

Eqs. (6.25), (6.26) are related with the superstring equations (6.13), (6.14) by $SO(2)$ rotations with the parameter

$$Cos\ \alpha = \sqrt{\frac{1+F(Z)}{2}}. \tag{6.27}$$

So, in general supergravity background the superstring and super–D1–brane are related by $SO(2)$ transformations whose parameter is constructed from the 10–dimensional counterpart $F(Z)$ of the generalized field strength $F^{(0)}$ of the super–D1–brane gauge field in the same way as in the flat superspace. However, in distinction to the case of flat superspace, $F^{(0)}$ is not constant, but is constructed from images of the axion and dilaton superfields on the worldsheet superspace (6.17). Thus the superstring and super–D1–branes are related by a *local* $SO(2)$ rotations, but with the parameter (6.27) dependent on the superspace coordinates through the mediation of the axion and dilaton superfield only (6.20). Such rotation appears as a compensated $SO(2)$ rotation [38, 39] when the $SL(2,R)$ (classical S–duality group) acts on the matrix constructed form axion and dilaton

$$K = e^{\frac{1}{2}\Phi} \begin{pmatrix} e^{-\Phi} & C_0 \\ 0 & 1 \end{pmatrix} \quad \in \quad \frac{SL(2,R)}{SO(2)} \tag{6.28}$$

Indeed, multiplying the matrix (6.28) by an element of global $SL(2,R)$ group

$$G = \begin{pmatrix} \frac{1}{c}\big(\frac{1}{\gamma} + \beta\big) & \beta \\ \gamma & c\gamma \end{pmatrix} \quad \in \quad SL(2,R) \tag{6.29}$$

from the left, one arrives at the transformation low for axion and dilaton superfield

$$GK(\Phi, C_0) = K(\Phi', C_0')H(G, \Phi, C_0), \tag{6.30}$$

where

$$H(G, \Phi, C_0) = \begin{pmatrix} Cos\ 2\alpha & Sin\ 2\alpha \\ -Sin 2\alpha & Cos\ 2\alpha \end{pmatrix}, \quad \in \quad SO(2) \tag{6.31}$$

$$Cos\ 2\alpha = -\frac{(c+C(Z))e^{\Phi(Z)}}{\sqrt{1+(c+C(Z))^2 e^{2\Phi(Z)}}} = -F(Z). \tag{6.32}$$

Eqs. (6.30), (6.31), (6.32) describe an induced $SO(2) = U(1)$ transformations acting on fields *with* $U(1)$ *charge equal to 2.* The $SO(2)$ rotation matrix acting on a doublet of fields which can be regarded as a complex field with $U(1)$ charge 1 has the form

$$\begin{pmatrix} Cos\ \alpha & Sin\ \alpha \\ -Sin\ \alpha & Cos\ \alpha \end{pmatrix} \quad \in \quad SO(2) \tag{6.33}$$

with

$$Cos\ \alpha = \sqrt{\frac{1 + Cos\ 2\alpha}{2}} = \sqrt{\frac{1 - F(Z)}{2}}. \tag{6.34}$$

The matrix (6.33) describes $SO(2)$ rotations of the fermionic supervielbein forms (cf. [43])

$$\begin{pmatrix} E^{\underline{\alpha}1\prime} \\ E^{\underline{\alpha}2\prime} \end{pmatrix} = \begin{pmatrix} Cos\ \alpha & Sin\ \alpha \\ -Sin\ \alpha & Cos\ \alpha \end{pmatrix} \begin{pmatrix} E^{\underline{\alpha}1} \\ \hat{E}^{\underline{\alpha}2} \end{pmatrix} \tag{6.35}$$

and can be used to relate super–D1–brane and superstring in the same manner as in the flat superspace.

To pass from super–D1–brane equations (6.25), (6.26) with the value of the field strength (6.17) to the superstring ones (6.18), (6.19) we need in the super–Weyl transformations (6.24) as well. They actually contain an important information. Indeed, if one performs such super–Weyl transformations in an action of 2–dimensional supersymmetric object in supergravity background, e.g. in the functional (4.1) taken on the surface of Born–Infeld equations (6.20), one arrives at the effective tension renormalized as follows

$$T'(Z) = T\sqrt{e^{-2\Phi} + (c + C_0)^2}. \tag{6.36}$$

If one considers Eq. (6.36) in a constant axion and dilaton background

$$\Psi =< \Psi >= const, \qquad C_0 =< C_0 >= const$$

and remembers that the Dirac quantization condition requires that both constants T and cT are integer valued (see e.g. [45])

$$T = p \ \in \ \mathbf{Z}, \qquad cT = q \ \in \ \mathbf{Z}, \tag{6.37}$$

one arrives at the famous formula for the (p, q) string tension [8]

$$T_{p,q} = \sqrt{p^2 e^{-2\Phi} + (q + pC_0)^2}. \tag{6.38}$$

Thus we can state that in general type IIB supergravity background

- superstring and super–D1–brane are described in a universal manner by the superembedding equation (6.12).
- They are related by the super–Weyl transformations (6.24) and induced $SO(2)$ rotations (6.35), whose parameter (6.34) is dependent on the worldsheet superspace coordinates through the mediation of the axion and dilaton superfields only (6.20). It is the compensating $SO(2)$ rotation [38, 39] for $SL(2, R)$ transformation (6.29) acting on the axion and dilaton superfields (6.30).
- Superstring equations appear as a singular limit $F^{(0)} \to -1$ of the super–D1–brane equations.
- The super–Weyl transform (6.24) reproduces the correct formula for the (p, q) string tension (6.38).

7 Conclusion and outlook

In this paper we worked out the generalized action principle for the fundamental superstring and super-D1-brane and derived the superfield equations of motion. In such a way we have proved that the superembedding equation (3.9) (or, equivalently, (1.4)) which describes an embedding of the worldsheet superspace $\Sigma^{(2|8+8)}$ with 2 bosonic and 16 fermionic directions into the $D = 10$ type IIB target superspace $\underline{\mathcal{M}}^{(10|16+16)}$, provides a universal, S-duality invariant description of the fundamental type IIB superstring and Dirichlet superstring (super-D1-brane).

In the case of flat target superspace the S-duality transformations manifest themselves in the $SO(2)$ symmetry of the superembedding equation. This continuous symmetry and the Weyl rescaling mix the fundamental superstring with a set of super-D1-branes 'marked' by a constant value of the (on-shell) field strength $-1 \leq F^{(0)} \leq 1$ of the worldvolume gauge field. Superstring with unit NSNS charge corresponds to the limiting value $F^{(0)} = -1$ of the gauge field strength.

We studied as well the case of general supergravity background and find that the $SO(2)$ rotation related the super–D1–brane and superstring has a parameter determined by the value of a constant and the dilaton and axion superfields. This is a compensating $SO(2)$ rotation for the S-duality $SL(2, R)$ transformation acting on 2×2 unimodular matrix constructed from the axion and dilaton superfields. The super–Weyl transform, which should be used together with the $SO(2)$ rotation on the way from the super–D1–brane to superstring, allows to reproduce the formula for (p, q) string tension (6.38) [8]. This supports the conclusion [11] that the (p, q) string can be associated with a D-brane action considered on the shell of Born–Infeld equation [11].

Note that one can not see any non–Abelian structure in such description, as it includes the description of just the bound state of p fundamental strings and q super-D1-branes, but not of the excitations over such bound state, which are described by non-Abelian $SU(q)$ SYM theory in the linearized approximation [9]. We hope that en effective Lagrangian description of a system of q coincident super-Dp-branes interacting through exchange by fundamental superstrings can be found by working out the recently proposed approach to the action of interacting superbrane systems [44].

The similar situation must occur for the superembedding equation, describing an embedding of the worldsheet superspace $\Sigma^{(6|16)}$ with 6 bosonic and 16 fermionic directions into $D = 10$ type IIB target superspace $\underline{\mathcal{M}}^{(10|16+16)}$. It should provide a universal description of the super–D5–brane, type IIB super–NS5–brane and super–KK5–brane (a D=10 type IIB Kaluza–Klein monopole). Such a study is of a special interest because the complete supersymmetric action are still unknown for the latter objects (see [7] and refs. therein for the bosonic actions).

We conclude with the following observation. Let us consider a system of several non-intersecting super-D1-branes and fundamental superstrings. Such system certainly preserves supersymmetry and, thus can be described in term of worldsheet superspaces. It is intriguing that we can consider them as one, but multiply connected worldsheet superspace characterized by one universal superembedding equation (1.4) or, equivalently, (3.9). Such superspace has several connected components

$$\Sigma^{(2|16)} = \oplus_k \Sigma_k^{(2|16)}.$$

The solution of the Born-Infeld equation (4.6) $dF^{(0)} = 0$ (or, more generally, (4.4)) provides each of the connected components $\Sigma_k^{(2|16)}$ with a specific value of the constant field strength of the worldvolume gauge field $F_k^{(0)}$ (or c_k (6.20)).

Thus one superembedding equation describes actually the system of several non-intersecting super-D1-branes and superstrings on the mass shall. The number of branes coincides with the number of connected components the worldvolume superspace is split on.

It is quite interesting to try to apply the superembedding approach for the system of several interacting branes: intersecting branes and/or open branes ending on other branes (see [27] for some results in this directions). However the problem one immediately meets is as follows. The language of worldvolume superspace used by the superembedding approach is certainly proper for a system which preserves supersymmetry. And this is not the case for the general system of interacting branes.

One of the possible ways is to develop the superembedding approach for the systems with 'soft supersymmetry breaking', e.g. to incorporate somehow terms dependent explicitly on the Grassmann coordinate of the worldvolume superspace into the superembedding equations. On the other hand, the most physically interesting system, which consists of several coincident D-branes interacting through exchange by fundamental superstrings, preserves the same amount of supersymmetry as the free branes. So, hopefully, one can find a description of such a system in a framework of some non-Abelian modification of the superembedding approach [6].

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[6] An existence of such approach is indicated indirectly by recent results on an existence of a non–Abelian generalization of the κ–symmetry [46].

Appendix A: Lorentz harmonics and equivalent forms of superembedding equations

As it was pointed out in Section 1.1, the superembedding equations (1.4) can be represented in the differential form notations as Eq. (1.7) [23]

$$\hat{E}^{\underline{a}} \equiv dZ^{\underline{M}}(\zeta)\hat{E}_{\underline{M}}^{\ \underline{a}}\left(\hat{Z}^{\underline{M}}(\zeta)\right) = e^{++}\hat{E}_{++}^{\ \underline{a}} + e^{--}\hat{E}_{--}^{\ \underline{a}}. \tag{A.1}$$

The bosonic forms $e^{\pm\pm}$ of the worldsheet supervielbein (1.6) have not been specified by any conditions. Hence, the components $\hat{E}^{\underline{a}}_{\pm\pm}$ have not been specified as well. The arbitrariness in the choice of $e^{\pm\pm}$ includes, in particular, general linear transformations

$$e^{++\prime} = \alpha e^{++} + \beta^{++}_{--}e^{--}, \qquad e^{--\prime} = \beta^{--}_{++}e^{++} + \rho e^{--}, \qquad det\begin{pmatrix} \alpha & \beta^{++}_{--} \\ \beta^{--}_{++} & \rho \end{pmatrix} \neq 0,$$

which imply the local $GL(2,R)$ redefinition of the 10-vector fields $\hat{E}^{\underline{a}}_{\pm\pm}(\zeta)$

$$\alpha\hat{E}^{\underline{a}\prime}_{++} + \beta^{--}_{++}\hat{E}^{\underline{a}\prime}_{--} = \hat{E}^{\underline{a}}_{++}, \qquad \beta^{++}_{--}\hat{E}^{\underline{a}\prime}_{++} + \rho\hat{E}^{\underline{a}\prime}_{--} = \hat{E}^{\underline{a}}_{--}. \tag{A.2}$$

These transformations can be used to achieve the light–likeness of both $\hat{E}^{\underline{a}}_{++}(\zeta)$ and $\hat{E}^{\underline{a}}_{--}(\zeta)$

$$\hat{E}^{\underline{a}}_{++}\eta_{\underline{ab}}\hat{E}^{\underline{b}}_{++} = 0, \qquad \hat{E}^{\underline{a}}_{--}\eta_{\underline{ab}}\hat{E}^{\underline{b}}_{--} = 0, \tag{A.3}$$

as well as normalization conditions

$$\hat{E}^{\underline{a}}_{++}\eta_{\underline{ab}}\hat{E}^{\underline{b}}_{--} = \frac{1}{2}. \tag{A.4}$$

The set of two light-like normalized vectors always can be completed up to the $SO(1,9)$ valued matrix

$$U^{(\underline{b})}_{\underline{a}} = \left(U^0_{\underline{a}}, U_{\underline{a}}^{\ J}, U^9_{\underline{a}}\right) = \left((\hat{E}_{--\underline{a}} + \hat{E}_{++\underline{a}}), U_{\underline{a}}^{\ J}, (\hat{E}_{--\underline{a}} - \hat{E}_{++\underline{a}})\right) \ \in \ SO(1,9). \tag{A.5}$$

To this end one introduces a set of 8 independent vectors $U^{\underline{a}I}$, $I = 1,\dots 8$ which are orthogonal to $\hat{E}^{\pm\pm}_{\underline{a}}$ and normalized

$$\hat{E}^{\underline{a}}_{\pm\pm}U^I_{\underline{a}} = 0, \qquad U^{\underline{a}I}U^J_{\underline{a}} = -\delta^{IJ}. \tag{A.6}$$

For convenience we can denote

$$\hat{E}^{\underline{a}\prime}_{++} \equiv \frac{1}{2}U^{\underline{a}--}, \qquad \hat{E}^{\underline{a}\prime}_{--} \equiv \frac{1}{2}U^{\underline{a}++}. \tag{A.7}$$

Than Eq. (A.5) acquires universal form (2.6)

$$U^{(\underline{b})}_{\underline{a}} = \left(U^0_{\underline{a}}, U_{\underline{a}}^{\ J}, U^9_{\underline{a}}\right) = \left(\frac{1}{2}(U^{++}_{\underline{a}} + U^{--}_{\underline{a}}), U_{\underline{m}}^{\ J}, \frac{1}{2}(U^{++}_{\underline{a}} - U^{--}_{\underline{a}})\right) \ \in \ SO(1,9). \tag{A.8}$$

Appendix B: On Superstring and super-D1-branes in curved superspace

Here we collect some useful formulae for superstring and super–D1–brane in general $D = 10$ type IIB supergravity background.

B1. Cartan forms and Maurer–Cartan equations in curved superspace

Cartan forms (2.20), (2.19) are invariant under the *global* Lorentz transformations of the vectors $U_{\underline{a}}^{\pm\pm}, U_{\underline{a}}^{I}$. However, when the superstring in curved superspace is considered, the harmonics are transformed by the *local* Lorentz group $SO(1,9)$

$$SO(1,9): \qquad U^{\underline{a}(b)\prime}(\zeta) = U^{\underline{c}(b)}(\zeta) L_{\underline{c}}{}^{\underline{a}}(\hat{Z}(\zeta)). \tag{B.1}$$

whose action on the supervielbein of type IIB supergravity is defined by

$$E^{\underline{a}\prime} = E^{\underline{b}}(Z) L_{\underline{b}}{}^{\underline{a}}(Z), \qquad L_{\underline{c}}{}^{\underline{a}} \eta^{\underline{cd}} L_{\underline{d}}{}^{\underline{b}} = \eta^{\underline{ab}}, \tag{B.2}$$

$$E^{\underline{\alpha}1,2\prime} = E^{\underline{\beta}1,2}(Z) \mathcal{L}_{\underline{\beta}}{}^{\underline{\alpha}}(Z), \qquad \mathcal{L}_{\underline{\gamma}}{}^{\underline{\alpha}} \tilde{\sigma}^{\underline{a}}{}_{\underline{\alpha\beta}} \mathcal{L}_{\underline{\delta}}{}^{\underline{\beta}} = \sigma^{\underline{b}}{}_{\underline{\gamma\delta}} L_{\underline{b}}{}^{\underline{a}}(Z). \tag{B.3}$$

Under the local $SO(1,9)$ group (B.1) the Cartan forms (2.19), (2.20) are transformed inhomogeneously. If we collect them in the antisymmetric (Lorentz algebra valued) matrix

$$\Omega^{\underline{ab}} \equiv U_{\underline{m}}^{\underline{a}} dU^{\underline{bm}} = \begin{pmatrix} 0 & \frac{f^{++J}+f^{--J}}{2} & -\frac{1}{2}\omega \\ -\frac{f^{++I}+f^{--I}}{2} & A^{IJ} & -\frac{f^{++I}-f^{--I}}{2} \\ \frac{1}{2}\omega & \frac{f^{++J}-f^{--J}}{2} & 0 \end{pmatrix} \tag{B.4}$$

then the transformation low can be written in a compact form

$$SO(1,9): \qquad \Omega^{(\underline{a})(\underline{b})\prime} = \Omega^{(\underline{a})(\underline{b})} + U^{\underline{c}(\underline{a})} (L^{-1}dL)_{\underline{c}}{}^{\underline{d}} U_{\underline{d}}^{(\underline{b})}. \tag{B.5}$$

To construct the forms which are invariant under (B.1) one can add to the original definition (B.4) the supergravity spin connection $w_{\underline{a}}{}^{\underline{b}}$ contracted with harmonic vectors

$$\tilde{\Omega}^{(\underline{b})(\underline{a})} = \Omega^{(\underline{a})(\underline{b})} + U^{(\underline{a})\underline{c}} w_{\underline{c}}{}^{\underline{d}} U_{\underline{d}}^{(\underline{b})} = U^{(\underline{a})\underline{c}} \left(dU_{\underline{c}}^{(\underline{b})} + w_{\underline{c}}{}^{\underline{d}} U_{\underline{d}}^{(\underline{b})} \right) \tag{B.6}$$

$$SO(1,9): \qquad \tilde{\Omega}^{(\underline{b})(\underline{a})\prime} = \tilde{\Omega}^{(\underline{b})(\underline{a})}.$$

(Actually this is a prescription for the construction of the so called 'gauge fields of non-linear realization' [38]).

Hence, the local Lorentz covariant coset vielbeine, $SO(1,1)$ and $SO(8)$ connections are

$$\tilde{f}^{\pm\pm I} \equiv f^{\pm\pm I} + (UwU)^{\pm\pm I} \equiv U_{\underline{a}}^{\pm\pm} (dU^{I\underline{a}} + w_{\underline{a}}{}^{\underline{b}} U_{\underline{b}}^{I}), \tag{B.7}$$

$$\tilde{\omega} \equiv \omega + \frac{1}{2}(UwU)^{--|++} = \frac{1}{2} U^{--\underline{a}} (dU_{\underline{a}}^{++} + w_{\underline{a}}{}^{\underline{b}} U_{\underline{b}}^{++}), \tag{B.8}$$

$$\tilde{A}^{IJ} = A^{IJ} + (UwU)^{IJ} \equiv U^{I\underline{a}} (dU_{\underline{a}}^{J} + w_{\underline{a}}{}^{\underline{b}} U_{\underline{b}}^{J}), \tag{B.9}$$

where $f^{\pm\pm I}, A^{IJ}, \omega$ are the original Cartan forms (2.20), (2.19).

The worldsheet covariant derivatives and, thus, worldsheet superspace torsion can be defined with the use of connections (B.8), (B.9) induced by embedding

$$t^{++} = De^{++} \equiv de^{++} - e^{++} \wedge \tilde{\omega}, \tag{B.10}$$

$$t^{--} = De^{--} = de^{--} + e^{--} \wedge \tilde{\omega}, \tag{B.11}$$

$$t^{+q} = De^{+q} = de^{+q} - \frac{1}{2}e^{+q} \wedge \tilde{\omega} + e^{+p} \wedge \tilde{A}^{pq}, \tag{B.12}$$

$$t^{-\dot{q}} = De^{-\dot{q}} = de^{-\dot{q}} + \frac{1}{2}e^{-\dot{q}} \wedge \tilde{\omega} + e^{-\dot{p}} \wedge \tilde{A}^{\dot{p}\dot{q}} \tag{B.13}$$

where $\tilde{A}^{pq}$ and $\tilde{A}^{\dot{p}\dot{q}}$ are s- and c-spinor representations for $SO(8)$ connection form $\tilde{A}^{IJ}$

$$\tilde{A}^{pq} = \frac{1}{4}\tilde{A}^{IJ}\gamma^{IJ}{}_{pq}, \qquad \tilde{A}^{\dot{p}\dot{q}} = \frac{1}{4}\tilde{A}^{IJ}\tilde{\gamma}^{IJ}{}_{\dot{p}\dot{q}}. \tag{B.14}$$

In (B.14) $\gamma^I_{p\dot{q}} \equiv \tilde{\gamma}^I_{\dot{q}p}$ are $SO(8)$ Klebsh–Gordan coefficients

$$\gamma^I_{q\dot{q}}\tilde{\gamma}^J_{\dot{q}p} + \gamma^J_{q\dot{q}}\tilde{\gamma}^I_{\dot{q}p} = \delta^{IJ}\delta_{qp} \qquad \tilde{\gamma}^I_{\dot{q}p}\gamma^J_{p\dot{p}} + \tilde{\gamma}^J_{\dot{q}p}\gamma^I_{p\dot{p}} = \delta^{IJ}\delta_{\dot{q}\dot{p}}, \tag{B.15}$$

$$\gamma^{IJ}_{qp} = \frac{1}{2}(\gamma^I\tilde{\gamma}^J - \gamma^J\tilde{\gamma}^I)_{qp}, \qquad \tilde{\gamma}^{IJ}_{\dot{q}\dot{p}} = \frac{1}{2}(\tilde{\gamma}^I\gamma^J - \tilde{\gamma}^J\gamma^I)_{\dot{q}\dot{p}}.$$

The $SO(1,9)_L \otimes [SO(1,1) \times SO(8)]_R$ covariant derivatives of the harmonic variables are defined with $SO(1,1)$ and $SO(8)$ connections and are expressed through covariant forms $\tilde{f}^{\pm\pm I}$

$$\mathcal{D}U^{++}_{\underline{a}} \equiv dU^{++}_{\underline{a}} - U^{++}_{\underline{a}}\tilde{\omega} + w_{\underline{a}}{}^{\underline{b}}U^{++}_{\underline{b}} = U^I_{\underline{a}}\tilde{f}^{++I} \tag{B.16}$$

$$\mathcal{D}U^{--}_{\underline{a}} \equiv dU^{--}_{\underline{a}} + U^{--}_{\underline{a}}\tilde{\omega} + w_{\underline{a}}{}^{\underline{b}}U^{--}_{\underline{b}} = U^I_{\underline{a}}\tilde{f}^{--I} \tag{B.17}$$

$$\mathcal{D}U^I_{\underline{a}} \equiv dU^I_{\underline{a}} + U^J_{\underline{a}}\tilde{A}^{JI} + w_{\underline{a}}{}^{\underline{b}}U^I_{\underline{b}} = \frac{1}{2}U^{--}_{\underline{a}}\tilde{f}^{++I} + \frac{1}{2}U^{++}_{\underline{a}}\tilde{f}^{--I}. \tag{B.18}$$

The covariant derivatives of spinor harmonics are

$$\mathcal{D}V_{\underline{\alpha}q}{}^{+} \equiv dV_{\underline{\alpha}q}{}^{+} - V_{\underline{\alpha}q}{}^{+}\tilde{\omega} + V_{\underline{\alpha}p}{}^{+}\tilde{A}^{pq} + w_{\underline{\alpha}}{}^{\underline{\beta}}V_{\underline{\beta}q}{}^{+} = \frac{1}{2}\tilde{f}^{++I}\gamma^I_{q\dot{q}}V_{\underline{\alpha}\dot{q}}{}^{-} \tag{B.19}$$

$$\mathcal{D}V_{\underline{\alpha}\dot{q}}{}^{-} \equiv dV_{\underline{\alpha}\dot{q}}{}^{-} + V_{\underline{\alpha}\dot{q}}{}^{-}\tilde{\omega} + V_{\underline{\alpha}\dot{p}}{}^{-}\tilde{A}^{\dot{p}\dot{q}} + w_{\underline{\alpha}}{}^{\underline{\beta}}V_{\underline{\beta}\dot{q}}{}^{-} = \frac{1}{2}\tilde{f}^{--I}V_{\underline{\alpha}q}{}^{+}\gamma^I_{q\dot{q}}. \tag{B.20}$$

The integrability conditions for Eqs. (B.16), (B.17), (B.18) indicate that the covariant Cartan forms satisfy the Maurer-Cartan equation

$$d\tilde{\Omega}^{(\underline{a})(\underline{b})} - \tilde{\Omega}^{(\underline{a})(\underline{c})} \wedge \tilde{\Omega}_{(\underline{c})}{}^{(\underline{b})} = U^{\underline{c}(\underline{a})}R_{\underline{c}}{}^{\underline{d}}U^{(\underline{b})}_{\underline{d}}. \tag{B.21}$$

It can be split on the following equations for the forms (B.7), (B.8), (B.9)

$$\mathcal{D}\tilde{f}^{++I} \equiv d\tilde{f}^{++I} - \tilde{f}^{++I} \wedge \tilde{\omega} + \tilde{f}^{++J} \wedge A^{IJ} = U^{\underline{a}++}R_{\underline{a}}{}^{\underline{b}}U^I_{\underline{b}}, \tag{B.22}$$

$$\mathcal{D}\tilde{f}^{--I} \equiv d\tilde{f}^{--I} + \tilde{f}^{--I} \wedge \tilde{\omega} + \tilde{f}^{--J} \wedge A^{IJ} = U^{\underline{a}--}R_{\underline{a}}{}^{\underline{b}}U^I_{\underline{b}}, \tag{B.23}$$

$$d\tilde{\omega} = \frac{1}{2}\tilde{f}^{--I} \wedge \tilde{f}^{++I} + \frac{1}{2}U^{\underline{a}--}R_{\underline{a}}{}^{\underline{b}}U^{++}_{\underline{b}}, \tag{B.24}$$

$$d\tilde{A}^{IJ} + \tilde{A}^{IK} \wedge \tilde{A}^{KJ} = \tilde{f}^{--[I} \wedge \tilde{f}^{++J]} + U^{\underline{a}I}R_{\underline{a}}{}^{\underline{b}}U^J_{\underline{b}}. \tag{B.25}$$

B2. Superstring

The derivative of the Lagrangian form of the fundamental superstring in general type IIB supergravity background is

$$d\hat{\mathcal{L}}_2^{IIB} = -2ie^{\frac{1}{2}\hat{\Phi}}\hat{E}^{++} \wedge \hat{E}^{-\dot{q}1} \wedge \hat{E}^{-\dot{q}1} + 2ie^{\frac{1}{2}\hat{\Phi}}\hat{E}^{++} \wedge \hat{E}^{+q2} \wedge \hat{E}^{+q2} + \tag{B.26}$$

$$+\frac{1}{2}e^{\frac{1}{2}\hat{\Phi}}\hat{E}^{++} \wedge \hat{E}^{--} \wedge \left(\hat{E}^{1-}_{\dot{q}}(V\nabla_1)^{+}_{\dot{q}}\Phi + \hat{E}^{2+}_{q}(V\nabla_2)^{-}_{q}\Phi\right) + \frac{1}{2}e^{\frac{1}{2}\hat{\Phi}}\hat{E}^{I} \wedge M_2^I + \propto \hat{E}^I \wedge \hat{E}^J.$$

where $\hat{M}^I_{2\ IIB}$ is the l.h.s. of the bosonic coordinate equation of motion for superstring in general type IIB supergravity background

$$\hat{M}^I_{2\ IIB} \equiv \hat{E}^{--} \wedge \tilde{f}^{++I} - \hat{E}^{++} \wedge \tilde{f}^{--I} + 4i\left(\hat{E}^{+q1} \wedge \hat{E}^{-\dot{q}1} - \hat{E}^{+2}_{q} \wedge \hat{E}^{-2}_{\dot{q}}\right)\gamma^I_{q\dot{q}} - \tag{B.27}$$

$$-\hat{E}^{++} \wedge \left(\hat{E}^{-\dot{q}1}\gamma^I_{q\dot{q}}(V\nabla_1)^{-}_{q}\Phi - \hat{E}^{-\dot{q}2}\gamma^I_{q\dot{q}}(V\nabla_2)^{-}_{q}\Phi\right) -$$

$$-\hat{E}^{--} \wedge \left(\hat{E}^{+q1}\gamma^I_{q\dot{q}}(V\nabla_1)^{-}_{\dot{q}}\Phi - \hat{E}^{+q2}\gamma^I_{q\dot{q}}(V\nabla_2)^{+}_{\dot{q}}\Phi\right) +$$

$$+\frac{1}{2}\hat{E}^{++} \wedge \hat{E}^{--}\left(H_{\underline{abc}}U^{++\underline{a}}U^{--\underline{b}}U^{I\underline{c}}e^{-\frac{1}{2}\Phi} - (U\nabla)^I\Phi\right) = 0.$$

Here

$$(U\nabla)^I \equiv U^{I\underline{a}}\nabla_{\underline{a}} \equiv U^{I\underline{a}}E^{\underline{M}}_{\underline{a}}(\hat{Z})\partial_{\underline{M}},$$

$$(V\nabla_2)^{+}_{\dot{q}} \equiv V^{+\underline{\alpha}}_{\dot{q}}\nabla_{\underline{\alpha}2} \equiv V^{-\underline{\alpha}}_{\dot{q}}E^{\underline{M}}_{\underline{\alpha}2}(\hat{Z})\partial_{\underline{M}}, \qquad (V\nabla_1)^{-}_{q} \equiv V^{-\underline{\alpha}}_{q}\nabla_{\underline{\alpha}1},$$

and $f^{\pm\pm I}$ are Cartan forms (B.7).

B3. Super–D1–brane

After some algebraic manipulations (but without any use of the supergravity constraints) one can present the external derivative of the Lagrangian 2-form (4.1) as

$$d\hat{\mathcal{L}}_2^{D1} = +\frac{e^{-\Phi}}{\sqrt{1-(F^{(0)})^2}}d\hat{\mathcal{L}}_2^{IIB} + \hat{R}_3 + e^{-\Phi}\sqrt{\frac{1-F^{(0)}}{1+F^{(0)}}}\hat{H}_3 + \tag{B.28}$$

$$+\frac{1}{2}\hat{E}^{++} \wedge \hat{E}^{--} \wedge \left(e^{\frac{1}{2}\Phi}F^{(0)}\ \hat{R}_1 - e^{-\frac{1}{2}\Phi}\sqrt{1-(F^{(0)})^2}\ d\Phi\right) + (d\mathcal{L}_2^{D1})_g + \propto E^I \wedge E^J,$$

where $d\hat{\mathcal{L}}_2^{IIB}$ is the external derivative of the Lagrangian 2–form of the fundamental superstring (B.26) and $(d\mathcal{L}_2^{D1})_g$ denotes the terms

$$(d\mathcal{L}_2^{D1})_g = \left(dQ_0 + e^{\frac{1}{2}\Phi}\hat{R}_1 - \frac{1}{2}Q_0 d\Phi\right) \wedge \left[e^{-\frac{1}{2}\Phi}\left(dA - \hat{B}_2\right) - \frac{1}{2}\hat{E}^{++} \wedge \hat{E}^{--}F^{(0)}\right] + \tag{B.29}$$

$$-\left(Q_0 + \frac{e^{-\frac{1}{2}\Phi}F^{(0)}}{\sqrt{1-(F^{(0)})^2}}\right)\left[\frac{1}{2}\hat{E}^{++} \wedge \hat{E}^{--} \wedge dF^{(0)} + F^{(0)}e^{-\frac{1}{2}\Phi}d\mathcal{L}_2^{IIB} + (1+F^{(0)})e^{-\frac{1}{2}\Phi}\hat{H}_3\right].$$

which determine dynamics of the gauge (super)field.

For the analysis of fermionic gauge symmetries and derivation of the fermionic equations (6.15), (6.16) it is sufficient to consider the derivative of the Lagrangian form $d\mathcal{L}_2^{D1}|_s$ taken on the surface determined by the superembedding condition (4.7), the gauge field constraint (4.3) and the algebraic (4.2). (This means, in particular that we can omit last two terms in Eq. (B.28)). After some straightforward but tedious algebraic manipulations with the use of supergravity constraints (6.2), (6.8), (6.9), (6.10) one obtains

$$
\begin{aligned}
& e^{\frac{1}{2}\Phi} d\mathcal{L}_2^{D1}|_s = \qquad\qquad (B.30)\\
& +i\sqrt{\frac{1+F^{(0)}}{1-F^{(0)}}}\hat{E}^{++}\wedge\left(\hat{E}^{2-}_{\dot{q}}+\sqrt{\frac{1-F^{(0)}}{1+F^{(0)}}}\hat{E}^{1-}_{\dot{q}}\right)\wedge\left(\hat{E}^{2-}_{\dot{q}}+\sqrt{\frac{1-F^{(0)}}{1+F^{(0)}}}\hat{E}^{1-}_{\dot{q}}\right)+\\
& -i\sqrt{\frac{1-F^{(0)}}{1+F^{(0)}}}\hat{E}^{--}\wedge\left(\hat{E}^{2+}_{q}-\sqrt{\frac{1+F^{(0)}}{1-F^{(0)}}}\hat{E}^{1+}_{q}\right)\wedge\left(\hat{E}^{2+}_{q}-\sqrt{\frac{1+F^{(0)}}{1-F^{(0)}}}\hat{E}^{1+}_{q}\right)+\\
& -\frac{1}{2}\hat{E}^{++}\wedge\hat{E}^{--}\wedge\left(\hat{E}^{2-}_{\dot{q}}+\sqrt{\frac{1-F^{(0)}}{1+F^{(0)}}}\hat{E}^{1-}_{\dot{q}}\right)\ V_{\dot{q}}^{+\underline{\alpha}}\left[(F+\frac{1}{2})\nabla_{\underline{\alpha}1}\Phi-(F-\frac{1}{2})\sqrt{\frac{1+F^{(0)}}{1-F^{(0)}}}\nabla_{\underline{\alpha}2}\Phi\right]\\
& -\frac{1}{2}\hat{E}^{++}\wedge\hat{E}^{--}\wedge\left(\hat{E}^{2+}_{q}-\sqrt{\frac{1+F^{(0)}}{1-F^{(0)}}}\hat{E}^{1+}_{q}\right)\ V_{q}^{-\underline{\alpha}}\left[(F-\frac{1}{2})\nabla_{\underline{\alpha}1}\Phi-(F+\frac{1}{2})\sqrt{\frac{1-F^{(0)}}{1+F^{(0)}}}\nabla_{\underline{\alpha}2}\Phi\right].
\end{aligned}
$$

Bosonic coordinate equation of motion for super-D1-brane in general type IIB supergravity background reads

$$
\begin{aligned}
& \hat{M}^I_{2(D1)} \equiv \frac{1}{\sqrt{1-(F^{(0)})^2}}M^I_{2(IIB)} - \hat{E}^{++}\wedge\hat{E}^{--}\left(U^{I\underline{a}}\nabla_{\underline{a}}\Phi - U^{I\underline{a}}R_{\underline{a}}e^{\Phi}\right) - \qquad (B.31)\\
& -\frac{1}{2}\hat{E}^{++}\wedge\hat{E}^{--}\left[\left(\sqrt{\frac{1+F^{(0)}}{1-F^{(0)}}}e^{-\frac{1}{2}\Phi}H_{\underline{abc}}+e^{\frac{1}{2}\Phi}R_{\underline{abc}}\right)U^{++\underline{a}}U^{--\underline{b}}U^{I\underline{c}}+\right] -\\
& -4i\gamma^I_{q\dot{q}}\left(\sqrt{\frac{1+F^{(0)}}{1-F^{(0)}}}\hat{E}^{+q1}\wedge\hat{E}^{-\dot{q}1}-\sqrt{\frac{1+F^{(0)}}{1-F^{(0)}}}\hat{E}^{+2}_{q}\wedge\hat{E}^{-2}_{\dot{q}}+\hat{E}^{+q1}\wedge\hat{E}^{-\dot{q}2}+\hat{E}^{+q2}\wedge\hat{E}^{-\dot{q}1}\right)\\
& +\hat{E}^{++}\wedge\left[\hat{E}^{-1}_{\dot{q}}\left(\nabla_{\underline{\alpha}2}\Phi+\sqrt{\frac{1+F^{(0)}}{1-F^{(0)}}}\nabla_{\underline{\alpha}1}\Phi\right)+\hat{E}^{-2}_{\dot{q}}\left(\nabla_{\underline{\alpha}1}\Phi-\sqrt{\frac{1+F^{(0)}}{1-F^{(0)}}}\nabla_{\underline{\alpha}2}\Phi\right)\right]\gamma^I_{q\dot{q}}V_q^{-\underline{\alpha}}+\\
& +\hat{E}^{--}\wedge\left[\hat{E}^{+1}_{q}\left(\nabla_{\underline{\alpha}2}\Phi+\sqrt{\frac{1+F^{(0)}}{1-F^{(0)}}}\nabla_{\underline{\alpha}1}\Phi\right)+\hat{E}^{+2}_{q}\left(\nabla_{\underline{\alpha}1}\Phi-\sqrt{\frac{1+F^{(0)}}{1-F^{(0)}}}\nabla_{\underline{\alpha}2}\Phi\right)\right]\gamma^I_{q\dot{q}}V_{\dot{q}}^{+\underline{\alpha}}=0.
\end{aligned}
$$

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